ENG 4002 Research Project

Solving the Leaky Tank Mystery

Final Report

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I. Acknowledgments

The group would like to extend our sincere appreciation to a set of people who have all played a significant role in the duration of this project. An individual paramount to this project, who is deserving of significant gratitude is the project supervisor Derek Abbott. Derek has supported the group with valuable advice and guidance on how to best approach and complete this project. We would also like to acknowledge Scott Letton, a member of the technical resource team who has provided beneficial advice and insight on how best to transfer our design ideas and objectives into a testable prototype.

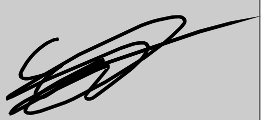
II. Declaration of Authenticity

The work, writing and figures found within this report are the intellectual property of the group. The only exceptions are information gathered from exterior sources, of which have all been cited and referenced.

III. Contribution Statement

The contribution statement includes a breakdown of the specific roles each individual group member has had throughout the duration of the project. The contribution statement for each of the group members illustrated below within table 1.

Table A.1: A detailed breakdown of the roles and contributions made by each group member.

Group Member	Role	Signature
Eric Tsoukatos (a1827083)	Conceptualisation, Data curation, Formal analysis, Investigation, Methodology, Project administration, Resources, Software, Validation, Visualisation, Writing – original draft, Writing – review & editing.	
Michael Stefani (a1825278)	Conceptualisation, Data curation, Formal analysis, Investigation, Methodology, Project administration, Resources, Software, Validation, Visualisation, Writing – original draft, Writing – review & editing.	

As can be seen above within the contributions table, there has been a uniform split of roles between group members, this is largely due to the small group size. Additionally, it is because many of the objectives and milestones to complete, each require a similar set of responsibilities.

IV. Executive summary

There exists much academic debate over a question that seems to be a simple physics-based puzzle: if a frictionless rail car filled with water develops a perfect vertical leak in an off-centre position as illustrated in Figure A.1, will the car move forward, backwards or not even move at all? This unknown effect is difficult to observe amongst other real-world phenomena, hence making it challenging to develop effective experimental techniques to settle this debate. However, investigation of the theory behind what seems to be a trivial matter offered a chance for a greater understanding of all forces and concepts involved.

Figure A.1: A diagram of a tank car which has sustained an off-centre leak.

Developing high sensitivity experiments for testing minute effects is also a broadly applicable skill for investigating the fine details of many physical models. This project also involved the development of a COMSOL multi-physics simulation model, which was utilised to verify theoretical calculations and experimental results. The findings computed by this simulative software clashed with the theoretical findings from the mathematical formulae. It was simulated that the tank would not experience any movement, whereas the tank was actually predicted to displace by 3.144 mm. As such, in order to truly depict the motion of this tank, a physical model was designed to be experimented with. The greatest challenge in testing this mystery physically is friction, therefore the design aimed to elevate the tank midair to eliminate the presence of friction. Upon experimental testing with this prototype, it was found that the tank will displace, contrary to what was computed by the COMSOL model. Further testing, proved that the size of the outlet diameter would also affect the movement of the tank with a 12 mm diameter outlet exhibiting 6.85 mm of displacement, and an 8 mm diameter outlet exhibiting 5.55 mm of displacement. Although the findings indicate that the flow of water directly affects the motion of the tank, it must also be stated that the presence of errors in testing do impede the accuracy of this statement.

V. Frontmatter

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1. Introduction

The leaky tank car mystery is a physics-based problem concerned with determining the motion of a tank car filled with water. Therefore, if this tank car is frictionless, and a leak is sustained in an off-centre position, what will the motion of this tank car be? Hence, this project focuses on exploring the parameters of the tank that affect the motion and how they correlate to a practical scenario.

1.1. Background

The leaky tank car mystery is a physics-based problem concerned with determining the motion of a tank car filled with water. The tank car begins at rest yet may begin to move once water begins draining from the tank through an off-centre hole. The leaky tank car mystery has been the subject of many investigations in the past. As such, numerous documents and papers have been written about this topic, however, what makes this mystery unique is that many of the experimental findings and results are not uniform across all papers.

1.2. Motivation

The leaky tank car problem has been approached and attempted in a number of ways by many researchers spanning over several decades. Despite all of this pre-existing research, attempts to determine the outcome of the leaky tank car have been largely limited to theoretical methods, with some believing that practical methods are not possible (McDonald 1991). This is what has driven the motivation for the project, to verify the solutions of theoretical analysis by simulating the problem with COMSOL and obtaining practical results through experiments.

1.3. Aims and Scope

The aim of this project was to be able to accurately determine and solve the mystery of the motion and behaviour of the leaky tank car, specifically how the car behaves when water begins draining from the body.

The project scope included physics simulation software such as COMSOL which was used to predict the flow and motion of liquids. Additionally, the physical design parameters of the tank were analysed in order to strike a balance between maximisation of motion of the tank, and feasibility. Furthermore, a physical model of the tank was also created in order to gain experimental results to analyse and compare with results obtained through simulated methods.

1.4. Objectives

Throughout the project numerous objectives were evaluated and explored. These objectives were used to guide the project to achieve the aim. The objectives for this project are:

OB1: Develop a simulation model that can determine the magnitude and direction of motion of the tank based on a range of parameters. This model is to be created and analysed in COMSOL. The model should have physical design parameters that are adjusted and analysed to investigate how each parameter affects displacement. Furthermore, the optimal parameters that have the greatest effect on motion of the tank modelled should be expressed. Results from the COMSOL model were used in conjunction with theoretical calculations to validate the results.

OB2: From the optimal parameters identified in OB1, two physical models can be constructed to investigate the effect of a single parameter. The physical tank was constructed to emulate the leaky tank mystery, however with adjustments made to be used for experimental testing. By constructing a tank with the same parameters as the simulated model, experimental results should replicate those computed by the COMSOL model.

1.5. Document Overview

The following report includes a wide variety of sections each used to convey information necessary to the completion of the project. The beginning of the document, includes important sections such as the executive summary, used to summarise the document and its outcomes. Additionally, the contribution statement, is used to identify the contributions to this final report from each group member. Following this is the introduction, equipped with a number of subsections that detail the context of the experiment such as background information, motivation behind the project, aims and objectives. From this, the literature review and theory portion of the document is introduced. The purpose of these sections were to conduct research on past reports and findings on similar projects allowing the group to gain a sound understanding of the project. Likewise, the methods part, again builds on from its predecessors, as now equipped with the required knowledge, a variety of methods used to complete the project have been listed. The technical chapters follow and are linked to the objectives section. Within these chapters are the detailed descriptions of how each objective was achieved and the factors that affected their completion. Following this is the recommendations for future work, a section used to identify what future developments of this project may include. Thus, this leads to the final sections of the document, a summary used to conclude the document and provide a synopsis of the discussion and outcomes, a set of references, from which the group has extracted knowledge from and an appendix including various miscellaneous writings such as coding scripts and apparatus datasheets.

2. Literature Review

The leaky tank problem or a minor variation of this topic has been the subject of numerous investigations in the past. Therefore, to grasp a better understanding of this topic, the laws associated, and what the expected results are, numerous literature reviews were conducted. One report titled the 'Motion of a leaky tank car' explored the theoretical motion that a tank car shaped as rectangular prism would experience if a square orifice was opened at one end of this tank car (McDonald 1991). To find out the motion of this tank car, a series of equations based on velocity and momentum were derived, these have been detailed and explored in the theory section (McDonald 1991). Theoretically, it is determined in relation to the law of conservation of momentum, as water is initially released, the tank car will gain some aspect of motion in one direction resultant of the momentum of water being pushed from the sides of the tank and out of the open orifice. However, since this flow rate decreases as a function of time, the forces applied by the tank walls are decreased resulting in a velocity reduction. As the velocity reaches its minimum, the momentum from the water flowing out of the tank is reabsorbed, thus reversing the direction of velocity of the tank car (McDonald 1991). The findings from this report state that a tank of length 20 m with cross section 25 m^2 and orifice opening sized 0.01 m^2 would theoretically result in the tank being moved at velocity of 0.06 m/s. Although a velocity was determined, it is likely that the force provided by this velocity would not overcome the magnitude of frictional forces between the tanker car and the ground it rests upon (McDonald 1991).

A similar study that corroborates the findings published in McDonald's report, explores the motion of a rail car filled with solid particles rather than a liquid. In this report it is determined that much alike the aforementioned study, the tank car will begin travelling in one specific direction for a period of time, before stopping and reversing its direction (Ekman 2019). Within this study, the theoretical water in the tanker is replaced with a set of balls that are alike the water, released through an off-centre opening (Ekman 2019). Further corroboration is included in an early document dating back to 1910 which analyses a tank of water with an efflux pipe resting on a frictionless plane (Wilson 1910). Again, the conclusion made is that the tank will begin moving in the direction opposite to the horizontal water flow, however there will be a point where this motion is reversed, and the tank will move back in the opposite direction (Wilson 1910).

The physical concepts present in this problem can also be applied to similar scenarios. As the leaky tank is essentially an object with constantly changing mass, so a similar effect would be expected in a coal car that is being filled from empty (MIT OpenCourseWare 2016). Through investigating initial and final momentum of the car, it is determined that the final velocity is a simple function dependent on the time taken to fill the car, as well as the initial parameters of the problem. However, for an emptying freight car, the momentum of the expelled particles must also be considered, and so the velocity of the tank at a given time is a more complex equation, however it is still a function of the change in mass (MIT OpenCourseWare 2016). Another analysis of a zig-zag shaped tank of emptying water determines the motion through multiple methods including conservation of momentum and energy. Through both analysis methods, it is determined that the motion will eventually reverse, and it is discovered that this reversed motion occurs when the rightmost section of the pipe is completely drained, such that the height of the water ≈ 0, however it is unknown how the water will behave at this point so the motion cannot be solved analytically (McDonald 2018).

A study published titled ‘Force, Momentum Change, and Motion’, explores the laws of motion at play and the formulae used to derive the motion of systems with a changing mass (Tiersten 1968). The findings made in this study are mostly applicable and can aid in better understanding the problem. One of the points of discussion from this study is that the equations of motion vary based on the type of system, as such some systems will be defined by force as a product of mass and acceleration, whereas others are defined by the momentum present within the system. However, it is stated that for the general case, a tank with a fixed volume and the possibility of admission/expulsion of particles, the equation of motion is based on the sum of momentum and momentum flux (Tiersten 1968), further analysis of this will be included in the theory subsection.

3. Methods

In preparation to complete this honours project, a number of different methods were theorised and detailed in order to best fulfill any objectives set. Therefore, throughout this section the adherence to these methods and how they were able to aid in the completion of necessary milestones will be detailed.

3.1. Initial Research

During the beginning of the year when the delegated project topics were distributed, each student in the group began preliminary research to gain a base understanding of the leaky tank problem, what it is and why does it occur. Following this, a set of literature reviews were conducted on documents and reports that had previously explored the leaky tank problem. This further research aided in strengthening our knowledge and understanding of the topic. The documents to review were found through a combination of searches on both Google and Google Scholar, in addition to being supplied by the group supervisor. The literature reviews illustrated within the section above, were each completed on a different set of reports. This was a necessary feature of the research since it provided an important perspective to be able to observe the differences between these reports, irrespective of the fact that they were exploring the same principle.

3.2. Design Conceptualisation

Following the literature review, the group had completed necessary research to understand a basic synopsis of the forces and concepts involved with the leaky tank problem. Therefore, from this understanding grew an initial conceptualisation of what experimental prototypes and features would be required to solve the leaky tank problem. This included ideas such as using a model tank car positioned on an air table or set of rails or hanging a tank of water in the air. After analysing the positives and negatives of these ideas, the group came to the conclusion that the prototype that should theoretically attain the most desired results is one that is suspended in the air from a support beam. This decision was made based on the idea that it will eliminate the variable of friction given that the tank will be suspended in the air, rather than sliding across a beam or table.

Since a basic understanding of how to design the experimental prototype had been developed, it was necessary to further add features to aid in the optimisation of the design. The primary feature requiring further tinkering was the outlet of the water. Given that the flow of water through this orifice is what generates the movement of the water tank, a uniform, constant release mechanism is required. In order to gain the most desired results, no external forces should be applied to the water tank, as such a remote mechanism to release water is needed. Therefore, possible mechanisms or techniques to release the water from the tank were researched until a component known as a smart tap timer was identified. This component allows an inlet of water to be controlled remotely, ensuring that the orifice opening for water release is constant in each experimental trial.

In order to build this prototype as seen in Figure 3.1, a set of geometric dimensions are first required for the size of the prototype. To calculate these dimensions, an analysis of the necessary parameters was done. COMSOL is also to be used to model and simulate the leaky tank. COMSOL is a multiphysics simulation software which can be used to simulate real-world designs (COMSOL 2024). Via utilisation of COMSOL the expected behaviour of the tank can be determined.

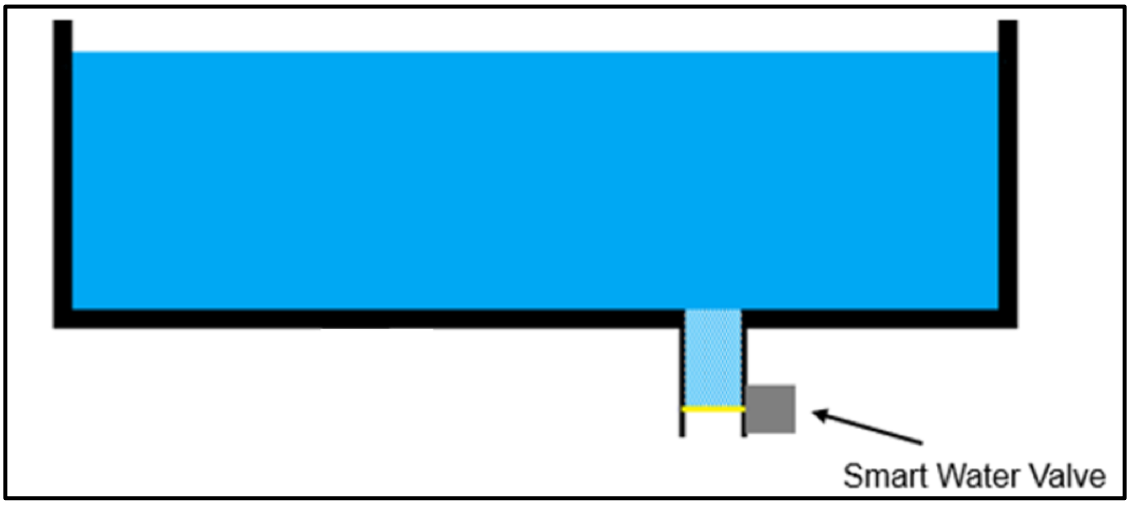

Figure 3.1: The theorised prototype to design, suspended in air. A smart water valve is also included for a constant water release system.

3.3. Testing and Experimenting

Once the prototype tank was built, experimentation and trials began. The water tank was filled with water to a measured amount and was suspended using fishing wire from a support beam. A remote-controlled smart tap timer was also used to control the water flow. In accordance with the law of conservation of momentum, the water being released from the tank should result in the tank being pushed to the side. The displacement of the tank was then measured using the combination of a laser measurer and mirror to magnify the displacement of the tank.

4. Theory

Some knowledge of various concepts relating to the momentum of the tank and flow of the fluid exiting the tank is required to understand the design considerations that must be carefully chosen. The basis of these principles and how they relate to the context of the problem are discussed in further detail.

4.1. Conservation of Momentum

Momentum is a property of an object relating to its mass and velocity, as a way to quantify the motion of the object. The conservation of momentum states that for a system that is not acted on by any external forces, the momentum is conserved in all directions (Hall 2021). In the context of this problem, the system consists of the tank, and the fluid exiting the tank. As the horizontal motion of the tank is the only direction that was investigated, the vertical motion can be neglected due to the fact that gravity acts on the tank and fluid in the vertical direction. Therefore, the horizontal momentum of the tank and fluid must always be conserved. This can be defined as $p_{initial} = p(t)$, where $p_{initial}$ represents the initial momentum of the system prior to the flow of liquid, while $p(t)$ denotes the instantaneous momentum of the system at any given time, t.

It is also vital to accurately setup the coordinate system used to define the model parameters and kinematic properties of the tank. As this is an investigation into a single horizontal dimension, the origin for this movement is set at the geometric centre of the tank, with the hole positioned in the positive direction. A diagram of this coordinate system in the context of the problem is shown in Figure 4.1.

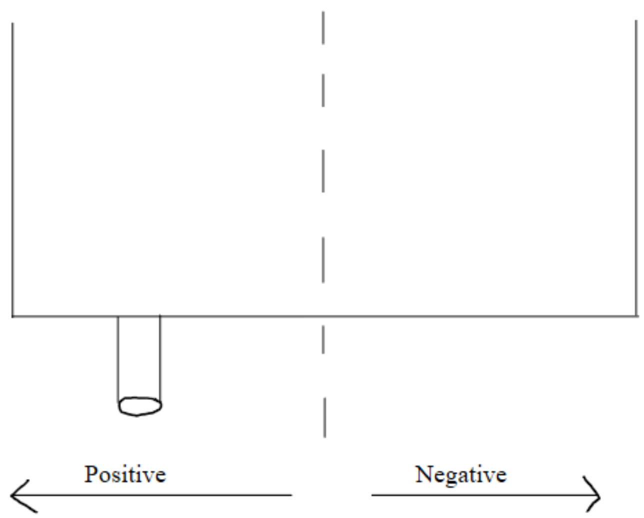

Figure 4.1: Sketch of the coordinate system for the leaky tank problem.

For cases where the mass of an object is changing, the relationship between momentum and the forces acting on the fluid in the tank can instead be written as (Tiersten 1968),

$$F = p + \Phi$$

Where F represents the force acting on the fluid in the tank, p represents the momentum of the entire system of the tank and fluid, while Φ is the momentum flux of the fluid leaving the tank.

From derivations based on the principles and equations of the conservation of momentum, the initial velocity of the tank is found to be (McDonald 1991),

$$V_{initial} = -\frac{2L}{t_{empty}} \times \frac{\mu}{1+\mu}$$

Where L is the distance of the hole from the centre of the tank, t_{empty} is the time it takes for the tank to empty, and μ is the ratio of total water mass to the mass of the tank, $\mu = \frac{M_0}{m}$. Therefore by increasing the mass of the water in the system, the ratio μ is increased, and the time it takes for the tank to empty also increases, resulting in a smaller initial velocity applied to the tank.

The acceleration, velocity, and displacement at any given time, t, can also be described by equations found from derivations of the conservation of momentum for the tank system (Esposito & Olimpo 2022).

$$a(t) = \frac{L}{{t_{empty}}^2} \frac{\mu}{1+\mu\left(1-\frac{t}{t_{empty}}\right)^2}$$

$$v(t) = -\frac{L}{t_{empty}} \frac{\mu}{1+\mu}\left[1-\frac{1+\mu}{\sqrt{\mu}}\left(\arctan\sqrt{\mu}-\arctan\sqrt{\mu}\left(1-\frac{t}{t_{empty}}\right)\right)\right]$$

$$x(t) = -L\left[\frac{\mu}{1+\mu}\frac{t}{t_{empty}} - \frac{1}{2}\left(\log\frac{1+\mu}{1+\mu\left(1-\frac{t}{t_{empty}}\right)^2} - 2\sqrt{\mu}\left(1-\frac{t}{t_{empty}}\right)\left(\arctan\mu-\arctan\sqrt{\mu}\left(1-\frac{t}{t_{empty}}\right)\right)\right)\right]$$

From these equations, it can be seen that when the hole is in the centre of the tank, $L = 0$, resulting in $x(t) = v(t) = a(t) = 0$. This means that the tank only moves due to the unsymmetrical nature of the system, resulting in reaction forces on the tank in order for momentum to be conserved.

Also, the final velocity of the tank can be determined for when $t = t_{empty}$. Applying this condition to the above equations results in the following equations that describe the motion of the tank.

$$a(t_{empty}) = \frac{L}{{t_{empty}}^2}\,\mu$$

$$v(t_{empty}) = -\frac{L}{t_{empty}}\left(\frac{\mu}{1+\mu} - \sqrt{\mu}\arctan\sqrt{\mu}\right)$$

$$x(t_{empty}) = L\left(\frac{1}{2}\log(1+\mu) - \frac{\mu}{1+\mu}\right)$$

Once again, the motion of the tank is heavily dependent on the mass ratio μ, and the location of the hole from the centre of the tank, L. While other parameters such as hole diameter, size of tank, and type of fluid are not explicitly mentioned in these equations, they all affect the emptying time, and mass ratio.

4.2. Laminar Flow

It was vital for the flow exiting the tank to be as stable as possible to prevent inconsistencies between multiple tests. This was achieved by designing the orifice of the hole in a way that ensures laminar flow is achieved. The turbulence of fluid flow is determined by the Reynolds number, which is calculated from the equation $Re = \frac{VD}{v}$, where V is the velocity of the fluid, D is the internal diameter of the pipe that the fluid is travelling through, and v is the kinematic viscosity of the fluid (Menon 2015). For flow to be laminar, this calculated Reynolds number must be less than 2000, which is achieved by decreasing the rate of flow or internal diameter of the pipe, or even a combination of the two. Achieving laminar flow increases the repeatability of the experiments, as turbulent flow would act as an uncontrolled variable, affecting the movement of the tank. The smoothness of the internal surfaces of the exit pipe and tap must also be considered, as any roughness can drastically increase the turbulence of fluid flow.

For a tank with a water level height of 0.20 m, the initial velocity of the water was found to be 1.98 m/s based on theoretical calculations. Since the kinematic viscosity of water is a constant (1.002×10^{-3} m^2/s), and the Reynold's number must be below 2000 to achieve laminar flow, the largest hole diameter that achieves laminar flow could be found. By using these values, the largest diameter that achieves laminar flow was found to be 1.0 m, but as the hole is well below this threshold, laminar flow will be achieved.

5. Technical Chapters

Since the concepts relating to this topic are now well understood, progress could be made towards the objectives, with the goal of working to the aim of the project. Progress was made by dividing the objectives into the discrete steps made in order complete the objective and create a set of outcomes. For this project, much of the initial planning for the achievement of these objectives were thought to be achieved in parallel, however OB1 must be completed first, before experimental testing can begin. This is because the physical parameters and dimensions of the leaky tank model are used in OB2 are based on the results of OB1. The first objective encapsulates the theoretical and simulation analysis of the leaky tank system. This was done through analysis of the design parameters that affect the tank movement and incorporation of a COMSOL model. In contrast, the second objective aimed to design a physical portrayal of the leaky tank system, such that it could be experimentally tested. The progress and steps taken to completing each of these objectives was discussed in the following sections.

5.1. Objective 1 – Theoretical and Simulation Analysis

Physics simulation software programs such as COMSOL and ANSYS, are a powerful tool that can simulate complex physics problems. To obtain meaningful results, it was necessary for the simulation to be as close to reality as possible. This means that the dimensions of the leaky tank model must be accurate, and any assumptions or simplifications made must be reasonable. The advantage of using these simulation programs over other theoretical methods is that the effect of each design parameter can be investigated much easier. To investigate how a design parameter affects the tank, the value can simply be adjusted to a different value, and the results can then be compared with the initial parameter.

5.1.1 Design Parameters

The motion of the tank was dependent on several design parameters. Some of these parameters relate to the size of the tank, while others relate to the size and location of the hole. The parameters and their description are displayed in Table 5.1, with their correlation to a physical model shown in Figure 5.1.

Table 5.1: List of all design parameters, with a description of how they relate to the tank.

Design Parameter	Description of Parameter
Width of Tank	The geometric dimension of the width of the tank
Depth of Tank	The geometric dimension of the depth of the tank
Liquid Level Height	The vertical distance from the hole to the surface level of the liquid
Location of Hole	The distance of the hole, from the centre of the tank
Diameter of Hole Orifice	The diameter of the hole at the off-centre location

Each of these parameters has been investigated through a theoretical analysis; in order to determine to what extent does each parameter affect the motion of the tank.

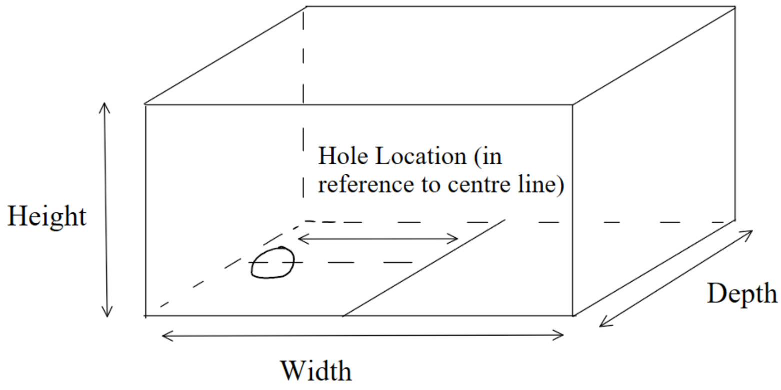

Figure 5.1: A basic sketch of the tank model indicating the nature of the dimensions used.

5.1.2 Parameter Analysis

Each of the key design parameters were analysed, to determine how changing each parameter affects the behaviour of the tank. This parameter analysis was done through using the displacement equation derived from the understanding of conservation of momentum, with code produced in MATLAB and shown in Appendix A. By investigating one parameter at a time, while all other parameters are fixed, it was directly determined how increasing or decreasing a particular parameter will affect the displacement of the tank. From the analysis of each parameter, the optimal values for the experimental setup were decided based on maximising the movement of the tank.

Width Analysis:
For the analysis of the width of the tank, four different values were selected: 0.20 m, 0.30 m, 0.50 m, and 1.00 m. All remaining parameters were kept constant, and so four curves were produced and plotted along the same axes in Figure 5.2. Examining this graph, it is clear to see that there was a general trend that the displacement increases as the width of the tank increases. This relationship appears to be almost linear, as increasing the width by a factor of 2, corresponds with the maximum displacement increasing by a similar factor. It is also worth noting that the time for the tank to empty also increases in a similar manner, which is not ideal for an experimental procedure, as this results in longer trials that take longer to record results from. An additional limitation to increasing the width of the tank is that excessively large tank size leads to complications due to the increased volume and weight of water, so the width is limited to what can be practically achieved.
From this analysis of the tank width, displacement was found to be maximised as width increases. However, increased tank width causes some difficulties with the experimental procedure, and as width is limited by what is practically achievable. Thus, a tank width of 0.30m was determined to be suitable for the experimental model.

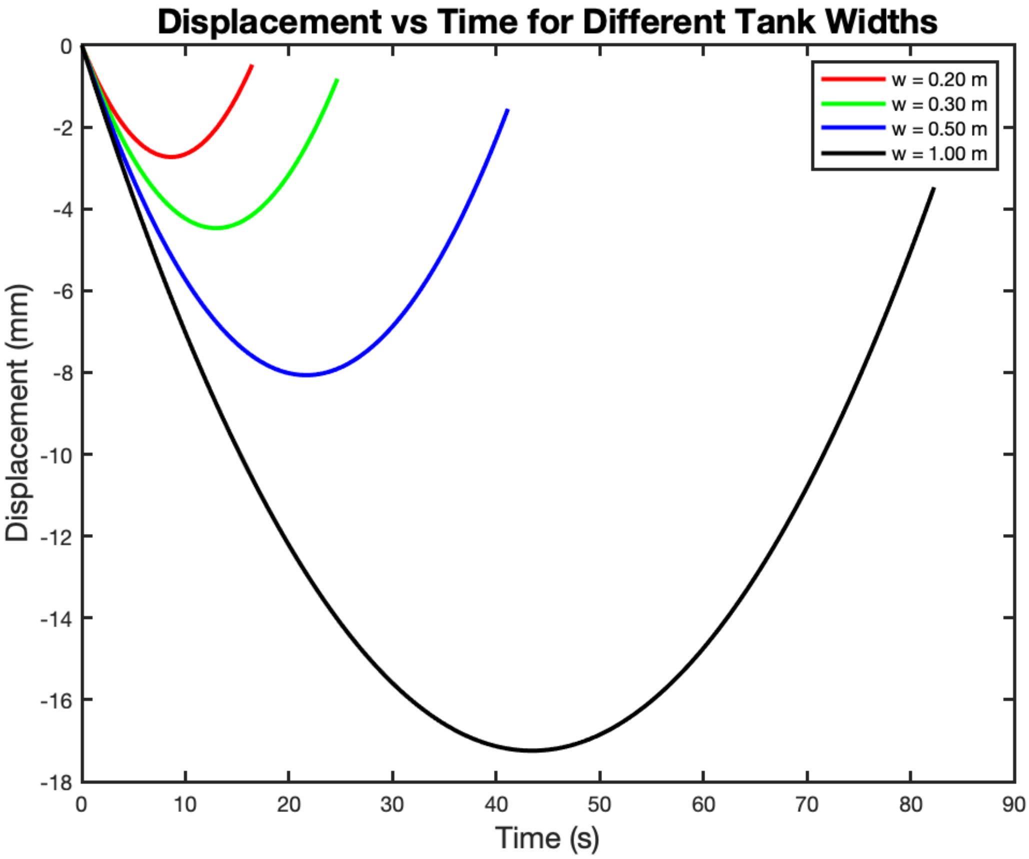

Figure 5.2: Graph of displacement against time for various tank widths.

Water Level Height analysis:
For the analysis of the water level height, five different values were selected: 0.10 m, 0.20 m, 0.30 m, 0.50 m, and 1.00 m. All remaining parameters were kept constant, and so five curves were produced and plotted along the same axes in Figure 5.3. This graph demonstrates the trend of the displacement increasing as the water level height increases. The water level also has an effect on the initial velocity of the water exiting the tank, where a higher water level height increases the water velocity, resulting in greater displacement and increased time to empty. The increase in maximum displacement between different water level heights is very minimal, so this parameter is not as essential as the other parameters to be maximised, since increasing the height also increases the time to empty and increases the volume of water in the tank. As the tank size is limited by practical constraint previously discussed in the width analysis, the height of the tank was set to be as high as possible without creating problems due to the weight of the water. A height of 0.20 m was deemed to be suitable for the leaky tank model, as it was optimal to favour an increase in other parameters that had a greater effect on displacement, such as the width of the tank.

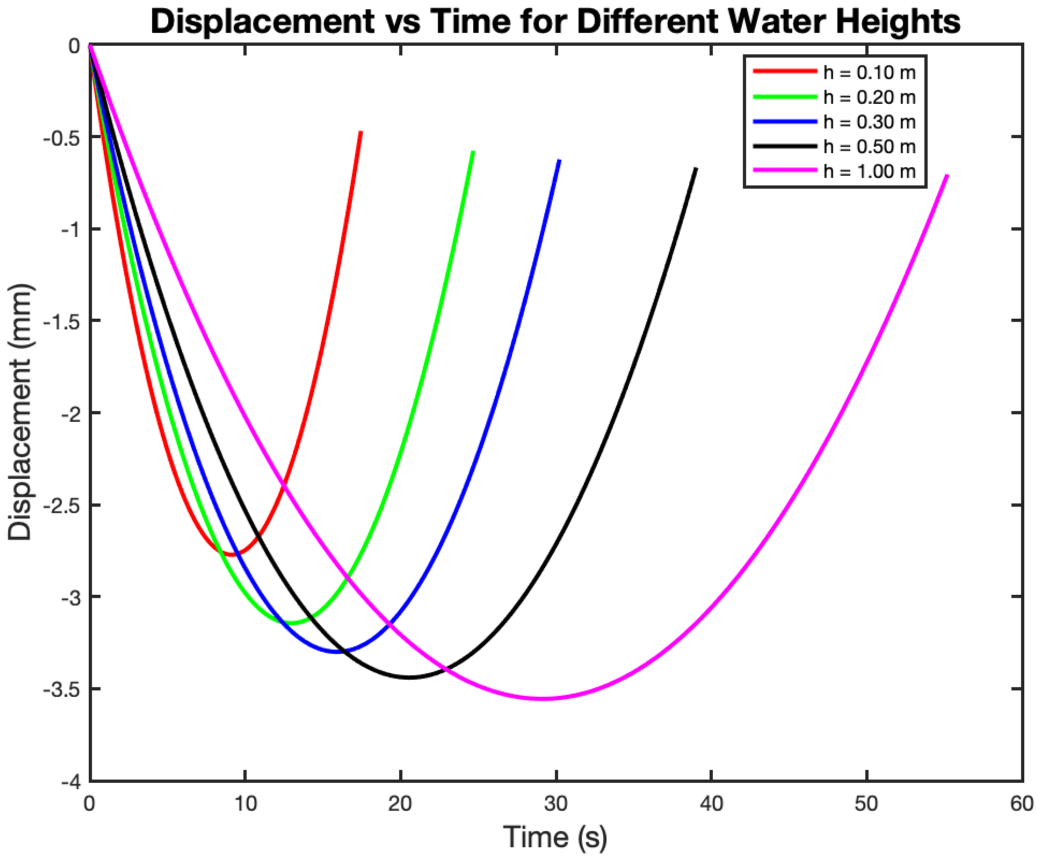

Figure 5.3:Graph of displacement against time for various water level heights.

Depth analysis:
For the analysis of the depth of the tank, five different values were selected: 0.10 m, 0.20 m, 0.30 m, 0.50 m, and 1.00 m. All remaining parameters were kept constant, and so five curves were produced and plotted along the same axes in Figure 5.4. This graph demonstrates the trend of the displacement increasing as the depth of the tank is increased. The relationship between maximum displacement and depth is not linear, as the effect that depth has on displacement decreases as depth is increased. This indicates that the displacement is limited by the remaining parameters rather than the depth, with the maximum displacement approaching a maximum possible value that is defined by this set of parameters. Additionally, the depth directly increases the weight of the tank through the capacity of water that it holds, so the depth is limited by the weight that the tank can practically support. As the depth was found to not be the most influential parameter and it is limited by physical constraints, it was set to value that allowed for a sufficient displacement, without significantly affecting the structural integrity of the tank. Therefore, a depth of 0.20 m was decided to be the most suitable value.

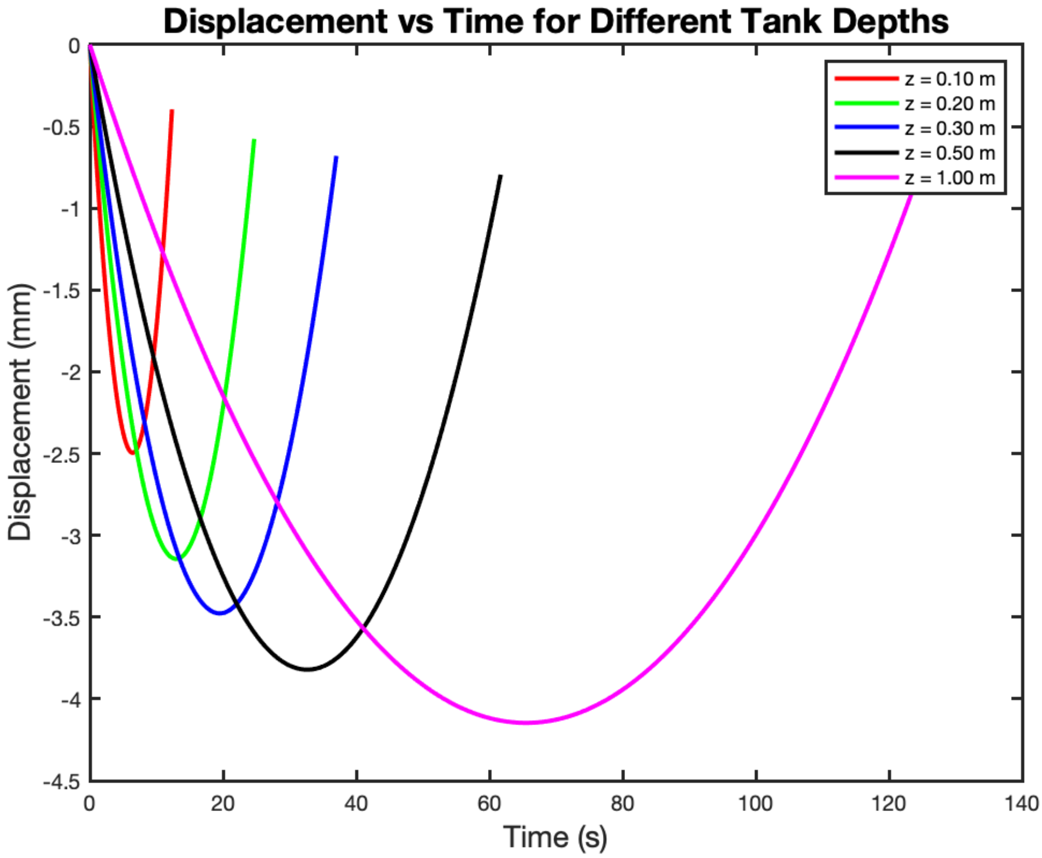

Figure 5.4: Graph of displacement against time for various tank depths.

Hole diameter analysis:

For the analysis of hole diameter, four different diameters were investigated: 0.01 m, 0.02 m, 0.03 m, and 0.04 m. The remaining parameters were kept constant, and four curves were produced and plotted on the same axes, shown in Figure 5.5. From this graph produced; it was evident that the diameter of the hole has a negligible effect on displacement. It does have a considerable effect on the time to empty though, where a smaller diameter drastically increased the time to empty. Thus, this analysis suggested a larger diameter is preferred, however this analysis failed to account for flow quality due to the size of the hole. Since laminar flow was desired to achieve repeatability in testing and reduce the presence of random errors, the Reynolds number must be kept as low as possible. The Reynolds number was kept as low as possible by keeping the hole diameter as small as possible. An additional consideration that had to be made was that the hole had to be able to attach to the device that allowed for flow to begin remotely, therefore the hole diameter was required to be of a standardised size such that it could easily connect to the device. While the diameter selected must have also been small enough to achieve sufficient flow quality, and large enough to have an ideal time to empty. From these requirements, a diameter of 0.025 m was selected as it was a standardised size that achieved desirable flow properties.

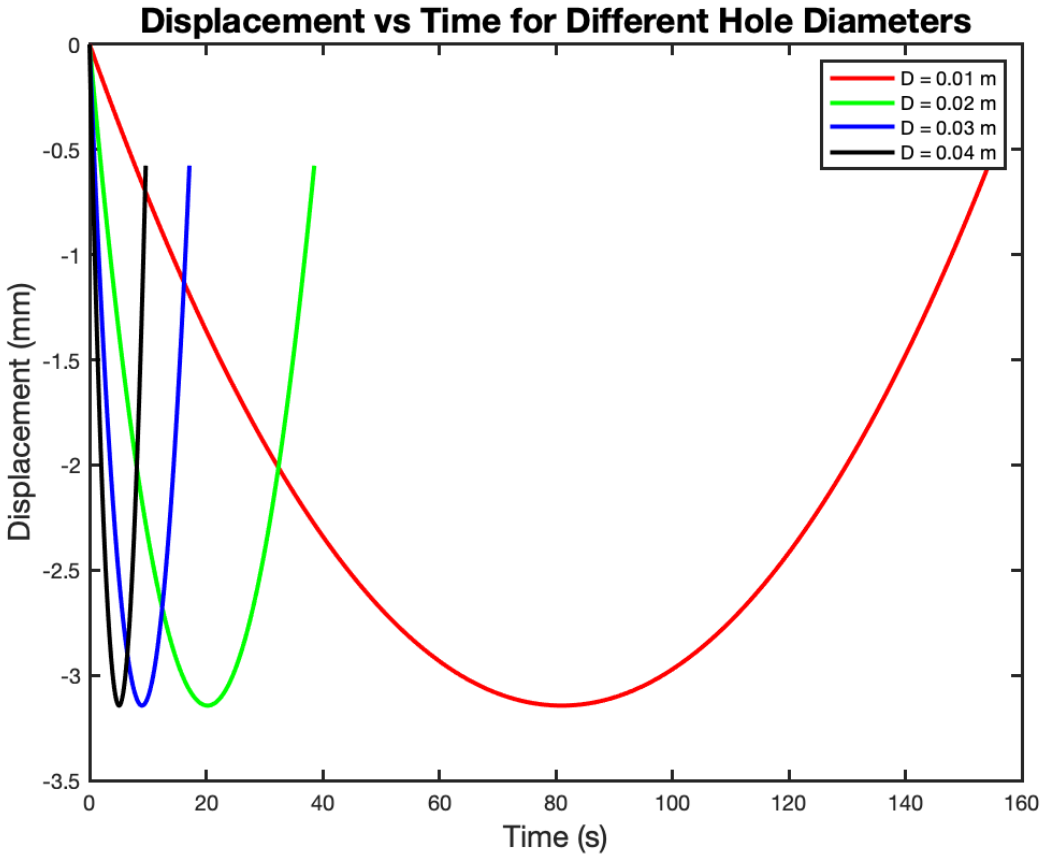

Figure 5.5: Graph of displacement against time for various hole diameters.

Hole Location analysis:
For the analysis of the hole location, five different values for the length over width ratio were selected: 0.00, 0.25, 0.50, 0.75, and 1.00. This ratio represents how far the hole is from the centre of the tank, where 2L/w=0 is at the centre and 2L/w=1 is at the end of the tank furthest from the centre. All remaining parameters were kept constant, and so five curves were produced and plotted along the same axes in Figure 5.6. From this graph, it is clear to see that the position of the hole has a large effect on the motion of the tank. For a hole at the centre of the tank, there was no displacement, as the forces in both directions are balanced. The displacement was maximised as the distance from the centre was increased, however this distance is limited by practical constraints such as the structural integrity of the tank and more obviously the hole cannot be further than half the overall width of the tank. The optimal location of the hole was found to be at the edge of the tank, since this is the point furthest from the centre, however any holes drilled into acrylic sheets should be located from the edge by at least twice the thickness of the sheet (KF Plastics 2023). While further recommendations are for the hole centre to be placed at least 1.5 times the diameter of the hole away from the edge of the acrylic sheet (KF Plastics 2023). Therefore, these guidelines were used to determine the hole location, with an additional factor of safety applied to ensure that the tank did not crack or break as the hole was drilled. The hole was decided to be placed 55 mm from the edge of the acrylic, for a ratio of 2L/w=0.633 and although this was more than the 1.5 times diameter recommendation, it was done to ensure the structural integrity of the tank was not compromised at all.

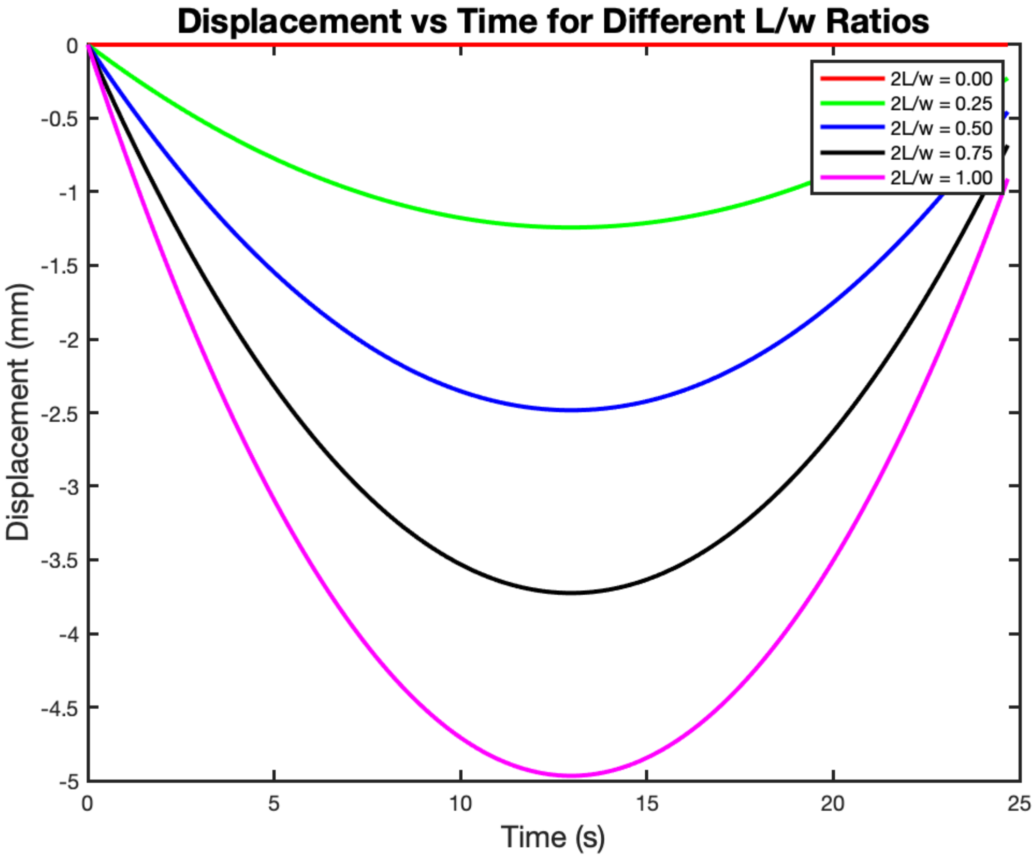

Figure 5.6: Graph of displacement against time for various L/W ratios.

5.1.3 Theoretical Calculations for Leaky Tank Model

By using the physical parameters of the tank obtained from the parameter analysis, shown in Table 5.2, theoretical calculations were then completed. These calculations are found in Appendix A and utilised the same equations as the parameter analysis, but rather than investigating the change in displacement for various values of each parameter, the equations were instead used to evaluate all of the properties that describe the motion of the tank. These properties include the time to empty, the initial velocity of the tank, the maximum displacement and the time at which maximum displacement occurs.

Table 5.2: Table of ideal dimensions to be used for the leaky tank model.

Design Parameter	Value Selected from Analysis
Width of Tank	0.30 m
Depth of Tank	0.20 m
Liquid Level Height	0.20 m
Location of Hole	0.095 m from centre (2L=0.633w)
Diameter of Hole Orifice	25 mm

From using the results outlined in Table 5.2, the calculations were then completed. The results of these calculations are shown in Table 5.3.

Table 5.3: Table of properties defining the expected behaviour of the tank.

Property of Motion	Value Selected from Analysis
Time to empty	24.68 s
Initial velocity of tank	-5.972E-5 m/s
Maximum displacement	-3.144 mm
Time to maximum displacement	13 s

5.1.4 COMSOL Design and Analysis of Leaky Tank Model

With reference to the parameter analysis, a set of dimensions for the designed leaky tank had been formulated. Using these dimensions, simulative methods such as use of COMSOL could now be used to generate a set of expected results. As such the group reached out to the IT team at the University of Adelaide regarding the procurement of a COMSOL license. Once the software had been acquired and set up within a computer, trials utilising the program began to devise the ideal setup to attain results. Upon research of the physics packages built-in to COMSOL, the fluid-structure interaction physics module was decided to be the best physics package available for modelling the displacement of the tank (COMSOL 2017). Although this was thought to be the ideal package to use, upon trying to test the model using this physics package the program would consistently output the same error, indicating that this physics module was a part of the structural mechanics module, of which had not actually been purchased within the licensing package that provided access to COMSOL.

Therefore, since the fluid-structure interaction module, along with numerous other physics modules were no longer an available option to use, the group improvised and adapted to design a model using the basic physics modules that were available. To begin, a new model was created, and the laminar flow physics module was added to the model. The next step was to create the virtual model of the tank with identical dimensions. This was done using two boxes, one larger and one slightly smaller, such that the difference of these two boxes was the tank sized with width 300 mm, depth 200 mm, height 200 mm and thickness 5 mm. A small cylinder was also included as the water outlet on the bottom of the tank. The acrylic material was then applied to the geometry built, as such this box is shown within Figure 5.7. Following this, water was selected as the fluid domain and was sized and assigned to fill the tank to its capacity.

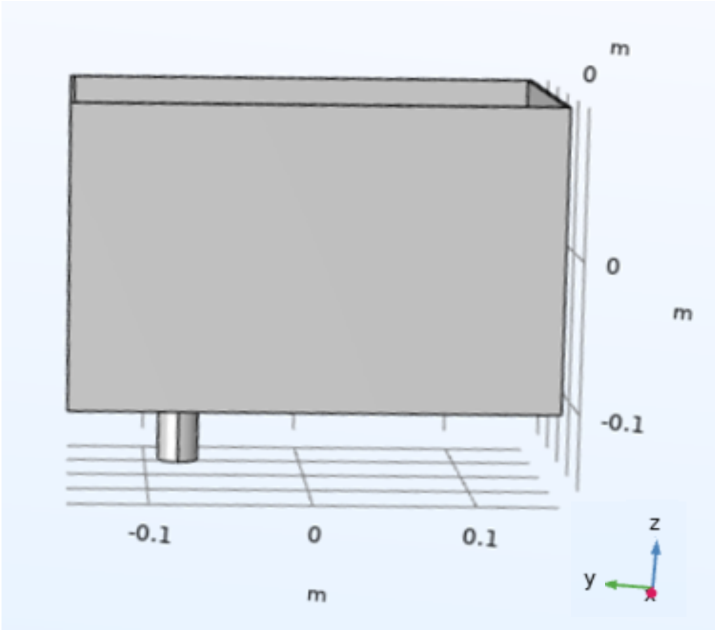

Figure 5.7: The leaky tank model designed in COMSOL.

Boundary conditions were also required to be defined, this includes definition of the water flow inlet and outlet, of which was simply completed via selection of each end of the pipe-cylinder created. It was also necessary to manually apply the pressure at the outlet as 0 Pa, assuming atmospheric pressure. Boundary conditions for each of the five tank faces were next to be applied. Since this tank is not fixed in any positions, and the tank is expected to move, time-dependent velocity conditions can be inserted based on the expected movement and velocity calculated within the parameter analysis. Therefore, each of the five walls are applied a velocity of -5.97×10^{-5} m/s in only the Y direction, since it is assumed that the tank will only experience motion in one dimension.

The model could then be meshed using structured hexahedral meshing, given the simplistic nature of the geometry. Following the mesh creation, a time dependent study was added and ran for 25 s, rounding up from 24.68 s, given that the tank should be emptied within this time, based on the characteristics of the tank evaluated within the theoretical calculations. Using this time dependent study, the solution was then computed and analysed. The main applicable result with relevance to this project that could be applied is the velocity profile of the tank. As such, a slice of the velocity of the tank was taken in the YZ plane to show how the simulated velocity profile would be affected. This slice was included below within Figure 5.8.

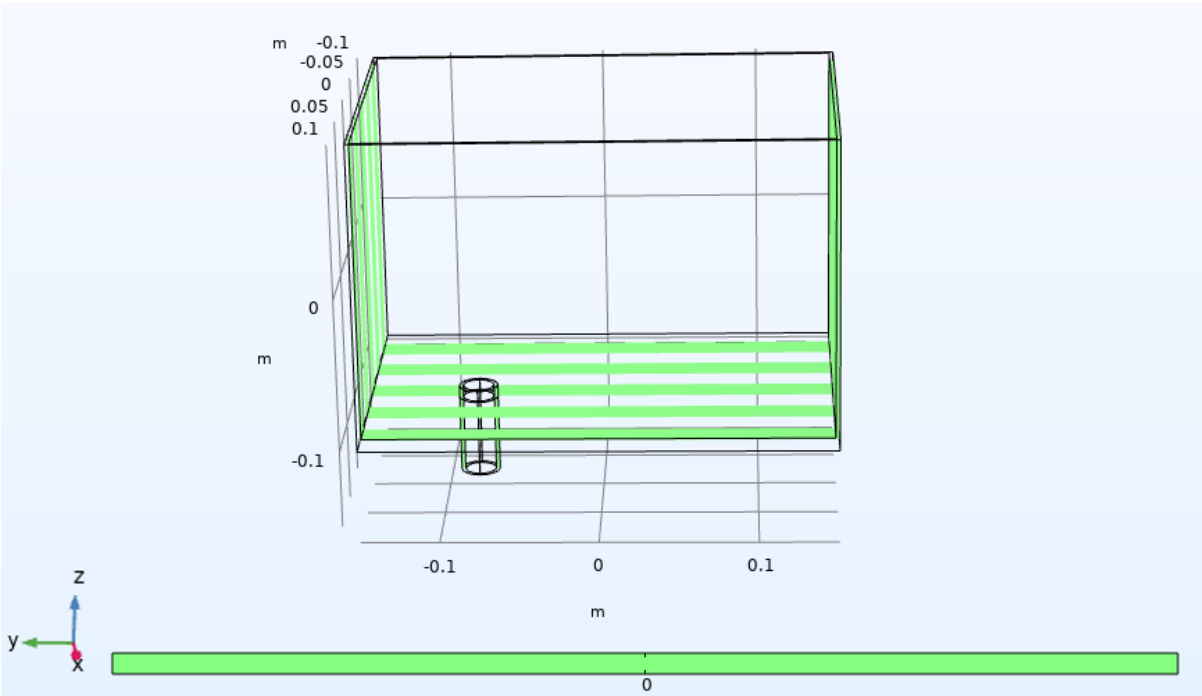

Figure 5.8: A slice of the tank model's velocity profile measured in the YZ-plane.

Evident within Figure 5.8, the velocity of the tank was generated to be zero, indicating that the tank would remain stationary, regardless of any water release. This is not the result that was expected, given that the theoretical calculations evaluated that the tank should move. Therefore, the model was adjusted with increased predetermined velocities to observe if the results would change however, they would continually define the model's velocity as zero. This suggests that the COMSOL simulation has an error within its set up. This problem may be due to the incorrect assignment of boundary conditions particularly regarding the inlet and outlet, or even the designation of prescribed velocity for the walls of the tank. In order to gain a proper set of data, numerous other methods would have been preferred such as use of the fluid-structure interaction physics module. However, due to persistent issues with the acquisition of COMSOL licensing this was not possible.

5.2. Objective 2 – Experimental Validation

Experimental procedures are utilised to connect the simulated results of COMSOL analysis to the practical and tangible outcomes of realistic scenarios. These experimental results also act to validate the COMSOL results, ensuring that the model and all assumptions made in COMSOL are reasonable. For the experiments conducted, it was necessary to consider all relevant details.

5.2.1 Experimental Planning

The experiment was to be designed and planned with careful consideration of everything that could have an impact on the results. By consulting with the technical resource team, input was provided by experts with vast knowledge of how best to construct the tank from the plans. From the initial meeting with the technical team, they were able to offer suggestions on alternative ways to construct the tank, which would provide either more accurate results, or come at a lower cost. One of the suggestions was to suspend the tank by a long length of material, so that the tank would be able to move more easily without the presence of friction. The team also indicated that based on the initial ideas, the setup should be simple to build and be able to yield a set of measurable results.

From incorporating the suggestions of the technical team to the initial design, the updated plan was to hang a rectangular tank from a structure that enables the tank to be suspended off the ground. The structure can be simply assembled using a beam of wood supported by two ladders at an equal height, such that the tank can suspended midair from as high a point desired. The wooden beam provides additional support and rigidity to the structure and allows for the connections between the beam and tank to be perfectly vertical. In doing this, the tank would be more susceptible to movement, resulting in less resistance to motion.

Another element of the project that had to be designed for was the leak in the tank. One of the main design challenges was to create a constant remote water release system. It was crucial that the release system was constant in order to maximise accuracy and repeatability of testing, this also ties into the remote requirement. Using a remote system to release water from the tank would prevent the tank from being subjected to any unnecessary external forces to the leaky tank that may arise from a manual water release system. As such, in order to build a tank that satisfies all requirements and design parameters, research of the materials and equipment available was necessary.

5.2.2 Materials and Equipment

There are a number of components necessary to use for the experiment. For each component it was important to consider the preferred properties to be able to accurately compare the options for each component. The main properties that must be considered include the cost, quality, reliability, and strength of the component in addition to any specific properties required of the component. The cost is extremely important as this project is assigned a budget based on the number of group members, and thus it was vital to stay below the budget in order to fulfill the aims of the project completely. The strength of the components was also a necessary consideration, since the tank must have been able to support its weight when filled with water, which may be quite significant given the volume of water the theorised tank would be able to hold. Therefore, if any component or aspect of the tank were to fail structurally, it would have catastrophic consequences on the experiment, due to the delays in purchasing of more components and fabrication of a new model for testing. Some components also had unique requirements, such as the remote release system which must be able to release water from the tank through remote capabilities. The following will detail the components required and their purpose in this project.

To complete this project experimentally, a tank is first required. Therefore, it was necessary to determine the material that this tank would be made of. There were numerous factors to consider when selecting a material for the walls of the tank, such as density, budget, transparency, water, shatter and corrosion resistance in addition to being machinable. It was important to ensure that the density of the material was kept low to prevent the tank from becoming too heavy given that it was being suspended in air with the addition of water weight. A transparent material was desired such that the behaviour of the tank could be tracked in relation to the amount of water in the tank at any given time. As the tank had a primary objective of holding water within, it was imperative that the material selected would not be prone to rusting or corrosion, as such only water-resistant materials were considered. The final requirement for the tank wall materials was to be machinable, since the material would be being cut to size and drilled within using power tools. In reference to these requirements metals were eliminated from contention due to their opaque nature, high density, and prone to oxidation (HowStuffWorks 2023). Wood was another material not considered also due to its properties, primarily based on its dimensional instability when exposed to water (University of Cambridge n.d). The remaining materials that were readily available at local retailers and passed all requirements included acrylic, polyethylene and polycarbonate. Of these options, polyethylene is not a strong candidate for holding large volumes as water (Vanderveer Industrial Plastics 2024), whereas polycarbonate exhibits a slightly increased density (Acme Plastics 2024), thus acrylic was selected as the ideal material for the tank walls.

Once acrylic had been selected for the tank, the next step was to identify a way of securing these acrylic pieces together in the desired formation of a water tank. Again, the main factor that affected selection was resistance to water, however, the bond between pieces also had to be strong enough to not fail under the loading caused by the weight of water held in the tank. Through research of bonding acrylics, three main solutions were apparent. These included acrylic cements, acrylic solvents or acrylic tape. Acrylic tape was the least optimal of these three, due to it exhibiting the lowest load-bearing capability of the possible options, the tape not creating a permanent bond, and degradation of the tape over time (Gytape 2023). Acrylic solvents were the next best option as they are fast setting and create a chemical bond between the acrylic sheets being connected, rather than attaching the sheets via an adhesive (Acme Plastics 2019). Similarly, acrylic cements also chemically fuse the two sheets together, however the cement creates a bond with far greater strength, as such acrylic cements such as Weld-on 16 evident in Figure 5.9 will be used to create the tank.

Figure 5.9: Weld-on 16, the acrylic cement used to bond the acrylic panels together (KF Plastics 2024).

The next challenge was to design how the tank would be suspended in the air. Aforementioned, it was stated that the tank could be hung from a level beam of wood. However, it was important to consider what would be used to suspend this tank from the beam, and how it would be attached to the tank. Attachment to the tank could be done in a multitude of different ways such as utilising eye hooks screwed into the corners of the containers, or using a cradle-based system to hold the tank from below. Both of these options required purchasing of extra components, thus, given that the budget for the group is already quite small, it was decided that holes could simply be drilled into the corners of the acrylic and the tank could be held up via a knotted hanging material around this point. Now that a hanging connection point had been decided, the hanging material was to follow. This could be done with string, wire, or a different thread such as fishing line. Although wire is the strongest of these materials, it is also the least flexible and would impede the tank from moving. Furthermore, fishing line was selected as the ideal choice given that thinner string/twine does not exhibit the required strength to hold the tank, whereas stronger strings are too thick and would not allow for as free a movement of the tank as fishing line. Specifically, high strength fishing line was used to keep the tank suspended in air.

Another feature of the tank that was incredibly important to design for was the release of water from the tank without physical input upon the tank or an attached component. This was done to prevent human error from interfering with results since it would introduce an external inconsistent force to the tank if water were to be released by hand. Therefore, water flow from the tank must be able to be controlled remotely. To do this, a set of gardening products were found that are available at most local retailers. These components are typically designed for gardening systems, and are remote controlled using an app, however, can be used for preventing or allowing water flow from the designed tank. These components included the Holman WX1 Tap Timer (Holman 2024), Orbit B-Hyve Tap Timer (Orbit 2024) and GARDENA Bluetooth Water Computer (GARDENA 2024). Since all products were able to complete the desired task, in order to minimise budgetary strain, the cheapest product of the three was selected, the Holman WX1 Tap Timer.

Once the tank had been put together and the testing was to begin, a way to measure the movement of the tank was required. Given that the tank designed was not expected to move at all according to the simulative analysis, and only very little according to the calculations made, a set of precise measuring equipment was required. As such use of a measuring tape was eliminated as an option due to its imprecision in use and since it could not measure movement of the tank simultaneously while the tank moves without impeding such movement. A laser measurer was also considered however, most laser measurers within our budget did not have a fine enough tolerance to identify the movement expected by the tank (SOLA 2024). As such a method involving use of a mirror and laser was chosen to magnify the actual displacement of the tank. This works by utilising displacement magnification, where despite a small movement of the mirror attached to the end of the tank, the reflected beam moves a larger distance and results in greater displacement of the reflected point.

Based on these considerations, the components and features of the tank have been identified and summarised below in table 5.4. Additionally, numerous options for each feature/component have been highlighted in green, this was done to indicate the choices selected for the prototype.

Table 5.4: List of all components required and their possible options, in addition to the components selected.

Component/Feature	Option 1	Option 2	Option 3
Material of tank	Acrylic	Polyethylene	Polycarbonate
Adhesive for building tank	Acrylic cements	Acrylic solvents	Acrylic tape
Method of connection	Eye hooks screwed into the tank	Cradle based support system	Holes drilled into the upper corners of the tank
Hanging material	Wire	String/Twine	Fishing line

Remote water release system	Holman WX1 Tap Timer	Orbit B-Hyve Tap Timer	GARDENA Bluetooth Water Computer
Measurement device	Laser measurer	Mirror and laser measurer	Measuring tape

Upon seeing what was required for construction and testing of the tank, the required purchases, their price and availability at local retailers were compiled into a document. This document, a risk assessment of the components being purchased, and documentation including verification from the group supervisor was then sent to the SET Store team who placed an order for the following items. Two Suntuf 900 x 600 x 5mm clear acrylic sheets, a hose attachment nozzle, as seen in Figure 5.10, one Holman WX1 Tap Timer and Wi-Fi hub, Figure 5.11 and two tubes of silicone. It should also be stated that the wi-fi hub was also required to be purchased along with the tap timer since the timer relies on connectivity to the wi-fi hub. The silicone was purchased for insurance protection to waterproof the edges of the tank where the acrylic cement was to be applied, in case the cement was not applied correctly. The final purchase made was an attachment to the outlet of the tap timer, an 8 mm outlet nozzle which can be used to reduce the outlet diameter. In doing this, the difference in behaviour of the tank based on outlets of different sizes can be observed.

Some other components were also required to allow the tank to function. For instance, the Holman WX1 Tap Timer attaches to a 25 mm thread, thus a 25 mm valve adaptor was used in order to facilitate the connection between the tank and the tap timer, seen in Figure 5.12. Other items included in the sections above that were not purchased, are items previously owned by group members. These items include high-strength fishing line, Weld-on 16, a high strength, clear, water resistant, type of acrylic cement used to bond acrylic sheets together (Acrylics Online 2024), and adhesive mirror panels.

Figure 5.10: 8 mm diameter nozzle tap attachment (Bunnings Warehouse 2024).

Figure 5.11: The Holman WX1 Tap Timer and Wi-Fi hub (Bunnings Warehouse 2024).

Figure 5.12: A 25 mm diameter valve adaptor (Bunnings Warehouse 2024).

5.2.3 Construction of the Experimental Model

Following purchase and pickup of the materials and components required for the experimental model, the group scheduled a meeting with the technical team. During the meeting, the team discussed the design of the model and the estimated timeframe for completion and pickup of the model from the technical team. However, dissatisfied with the estimated timeframe, the group members decided to mitigate the waiting process and instead build the experimental model themselves using the materials purchased and resources available.

Evaluated in the parameter analysis, the experimental tank is to be an open-top rectangular prism with dimensions 300 x 200 x 200 mm. As such, to begin construction of the model, the acrylic sheet was measured and divided into five segments, each segment based on one of the five faces of the tank. Therefore, the required segments were as follows, three 300 x 200 mm panels, and two 200 x 200 mm panels. Whilst hand tools were initially used for cutting the acrylic, it was experienced that when cutting acrylic close to an edge with an unbalanced side, the acrylic panel would likely fracture due to the excessive force of the handsaw pulling the acrylic from the sheet. Thus, after this learning experience, it was decided that power tools such as a pull-down circular saw would be used to cut the panels instead.

Once the panels had been cut, the next step was to drill the necessary holes into the acrylic before it was cemented together. This included five holes in total, two small holes on the top corners of each of the larger side panels as seen in Figure 5.13 and one hole on the bottom panel displayed in Figure 5.14. The small holes are designed for the fishing line to feed through to hold and suspend the tank midair, whereas the bottom hole is designed for the 25 mm valve adaptor to be cemented to thus allowing water to travel out and through the tap timer. These holes were made using an electric hand drill and simply drilling through the acrylic, however the larger hole on the bottom panel used a much larger drill bit, hence a greater sized hole. When drilling the holes, it was also imperative to follow a specific set of guidelines to minimise the chance of cracking the acrylic. This rule was to ensure that the distance between the edge of the acrylic panel and the hole was 1.5 times the diameter of the hole (KF Plastics 2023). Therefore, given that the diameter of the drill bit being used to drill the larger hole was 7/8 inches (22.2 mm), the hole was drilled 40 mm from the edge of the panel, thus in compliance with the recommended guidelines, in addition to the inclusion of a safety factor. In doing this, no fractures occurred whilst drilling the holes into the panels. Since the larger hole drilled is only of diameter 7/8 inches, the hole must be expanded by 1/8 inch in order to fit the 25 mm diameter inlet valve adaptor. To do this, the panel was simply set into a vice and the hole was filed down using a round hand file until the adaptor could fit. Although some sides were filed more than others creating gaps between the acrylic panel edges and the valve adaptor, this was a simple fix since the acrylic cement was able to set in the gaps and create a flush surface.

Figure 5.13: An acrylic panel exhibiting two small holes on the top corners (circled in red) used to loop fishing line though to hold the tank.

Figure 5.14: An acrylic panel exhibiting one larger hole (circled in blue) that will be used to fit the valve adaptor.

Now that all panels had been machined to their required amounts, the tank was ready to be assembled. All panels of the tank were first assembled together using tape to ensure that all the panels fit together well to create the desired shape. In verifying that the assembled tank was of the desired size and fit, adhesion of the panels was ready to commence. This began with the taping up of the first two pieces pressed against one another creating a right angle at the edge. Once confirmed that the two pieces were perpendicular to one another using a square precision tool, the Weld-on 16 was applied at the seam of the two acrylic panels. These panels were then left for fifteen minutes placed in a stand to allow the acrylic cement to initially set between the two panels. Following this, a third panel was attached with tape and held square whilst applying the Weld-on 16. Again, the panels were left alone such that the cement could set. The fourth panel was then attached and left to rest, and then the final panel was bonded also. The final product still taped together, was then left to set for 24 hours. Following the 24-hour setting period, the valve adaptor was also bonded to the hole on the bottom panel of the tank using Weld-on 16, the tank as seen in Figure 5.15 was then left to set for another 24 hours. Additionally, a small adhesive mirror panel was adhered to the side of the tank, evident in Figure 5.16. This was done to aid in measuring the displacement of the tank when testing, through use of a laser measurer.

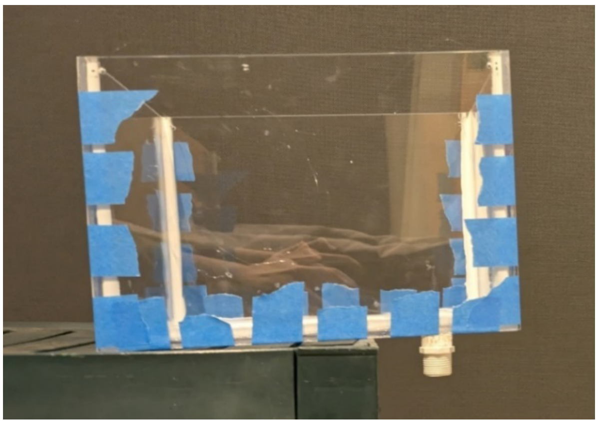

Figure 5.15: The tank with all sides taped together left overnight for the acrylic sheets and valve adaptor to bond.

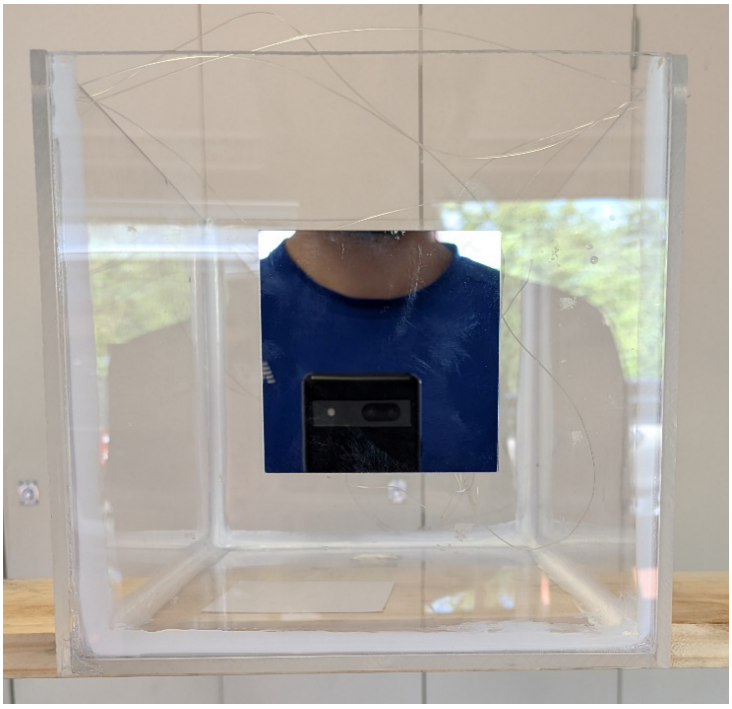

Figure 5.16: The mirror panel applied to the side of the tank for aid in measurement purposes.

The final step of construction was then ready to be complete, this involved applying a light coating of silicone to all edges of the tank where water may try to make its way in. Although the Weld-on 16 should already prevent this from occurring, the choice to silicone the tank was made as an insurance against the weld-on not being applied uniformly across the edges. Therefore, once this coat of silicone was applied, the tank was left for 72 hours giving the silicone the necessary time required for it to cure. The construction of the experimental tank had now been completed as seen in Figure 5.17 and was ready to be used in testing.

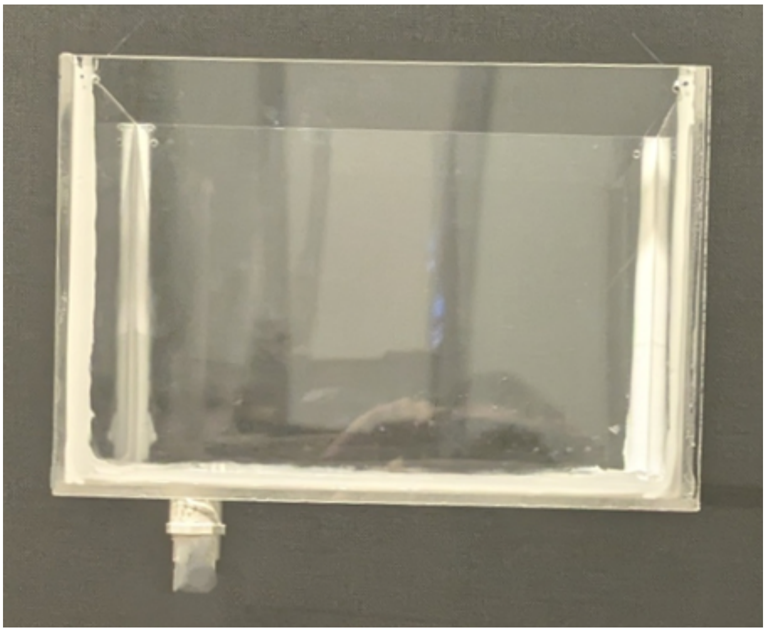

Figure 5.17: The completed experimental tank with silicone applied, now ready to be used in testing.

5.2.4 Experimental Testing

Upon completion of the leaky tank construction, experimental testing could now begin. When setting up an environment for testing, several considerations had to be taken into account. Through combination of the small size and suspension mid-air, the tank is prone to swaying. Even if only a minute force is applied, the tank will continually sway for many minutes until it stops naturally or via an external force. Therefore, any presence of wind would sway the tank and have a significant impact on the results. To combat this, the experiment was completed indoors, with all airflow to any external environments restricted. The length of the fishing line suspending the tank from the wooden beam was also held constant on either side of the tank. This was done to ensure that the motion of the tank was only influenced by the water being released.

To begin testing, the Holman WX1 Tap Timer was screwed into the valve adaptor bonded to the tank. Two equidistant lengths of fishing line were cut and looped within two of the smaller holes of the larger panels. Both ends of the line were then tied together creating a loop of fishing line that travels through the two smaller holes that directly face each other. The next step was to place a wooden beam level across two ladders positioned at the same height. The tank could then be suspended from this beam by simply navigating the beam through the two fishing line hoops such that the loops would lay upon this beam and support the tank. The tank was then ready to be filled with water. Initially the tank was first filled to its maximum volume, however in doing so, this caused the fishing line to snap. Fortunately, the tank was caught by one of the group members preventing any damage to the tank. As such, after reapplying the fishing line loops, the tank was instead only filled to half the capacity of the tank, to minimise the possibility of damaging the tank. Whilst waiting for the tank to stop swaying and become stationary, a separate ladder was set up at the height of the mirror panel upon the tank. This ladder was used to hold the laser measurer in a constant position, pointed at the mirror and reflected onto the wall. The laser was measured to be 0.887 m away from the edge of the tank, and the distance from the tank to the reflected point was 2.202 m. Utilizing trigonometric ratios, the angle of incidence of the laser pointer on the mirror was found to be 22.4°, resulting in an angle of 135.2° between the incoming laser ray and the reflected laser. Tape was also used on the area where the laser was reflected, as this allowed for measurements to be overlayed behind the reflected laser to understand the displacement measured along the wall due to the actual displacement of the tank.

With the tank filled with water and the laser measurement stand set up, experimental testing was ready to begin. Images of the setup can be observed in Figures 5.18 and 5.19. Since testing was now ready to begin, a camera was set up in front of the wall where the lasers were being reflected to record and capture the behaviour of the laser pointer as water is released from the tank. The water was then released via manual input controlling the tap timer, and simultaneously the movement of the tank was recorded through the behaviour of the laser. This testing was then repeated and captured again, however, this time the 8 mm outlet nozzle was equipped to the tap timer.

Figure 5.18: The experimental set up, including the tank being suspended from a wooden beam and a container placed below the tank to catch the water released.

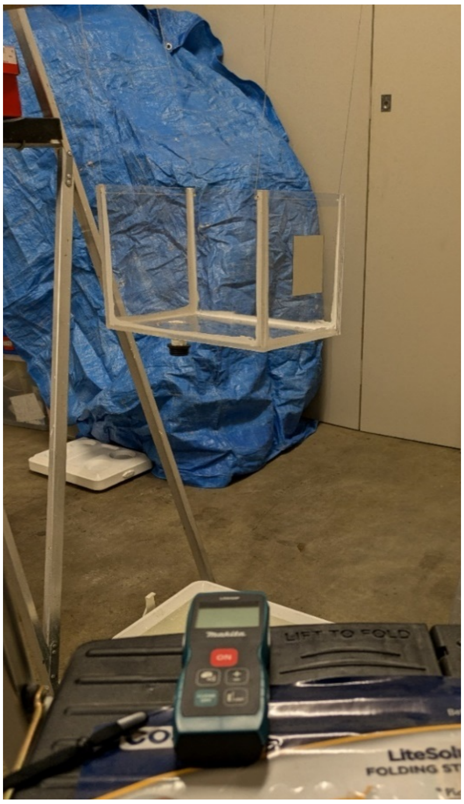

Figure 5.19: The experimental set up of the laser measurer mounted upon a ladder pointed towards the mirror panel on the leaky tank.

5.2.5 Results

Upon experimentally testing the leaky tank, several results were obtained directly from the experiment, and some were derived through further calculations. The tank and laser pointer were setup as discussed previously, with the laser pointed at the tank in such a way to measure the actual displacement of the tank through the magnified displacement on the wall, as seen in Figure 5.20. Prior to the experiment, the measured distance of the laser ray from the laser source to the tank, and then to the reflected point on the wall was measured to be 3.089 m in total.

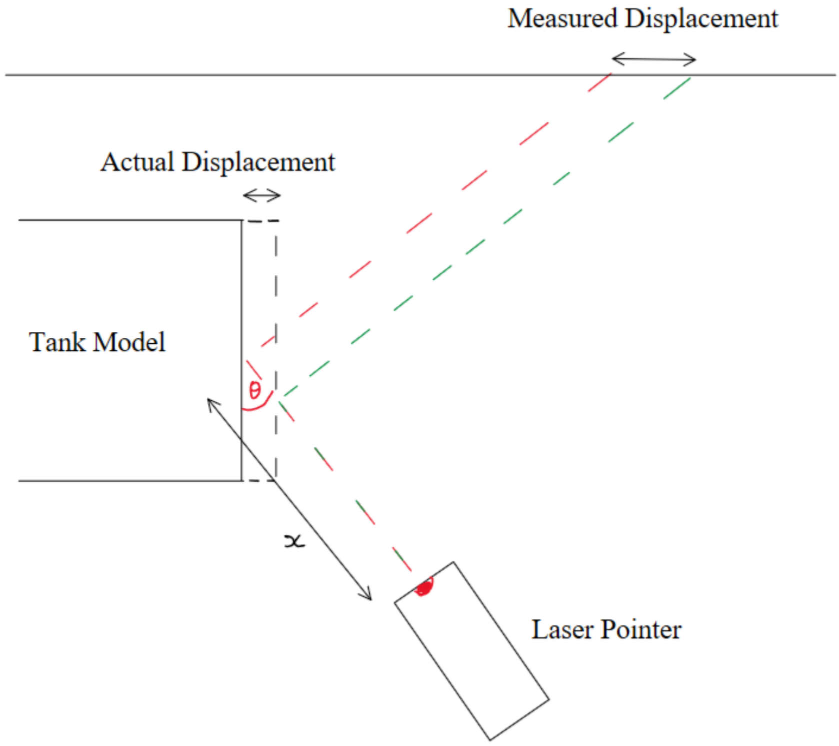

Figure 5.20: Top-view diagram of the experimental setup used to measure the displacement of the tank.

During the experiment at the maximum displacement, the distance of the same scope was measured to be 3.101 m. Although this represents an increase of 12mm in length, this result is a magnification of the true displacement of the tank due to the setup of the experiment. Additionally, the difference between the red dot representing maximum displacement and the larger black dot representing the starting position prior to motion, from Figure 5.21 was measured to have a difference of 2.1 cm. From using these measured values and through using trigonometry, the angle of reflection, orthogonal to the mirror, was found to be

$$\beta = \cos^{-1}\frac{\Delta L}{\Delta d}$$

$$\Rightarrow \beta = \cos^{-1}\frac{3.101\ m - 3.089\ m}{0.021\ m} = 55.2°$$

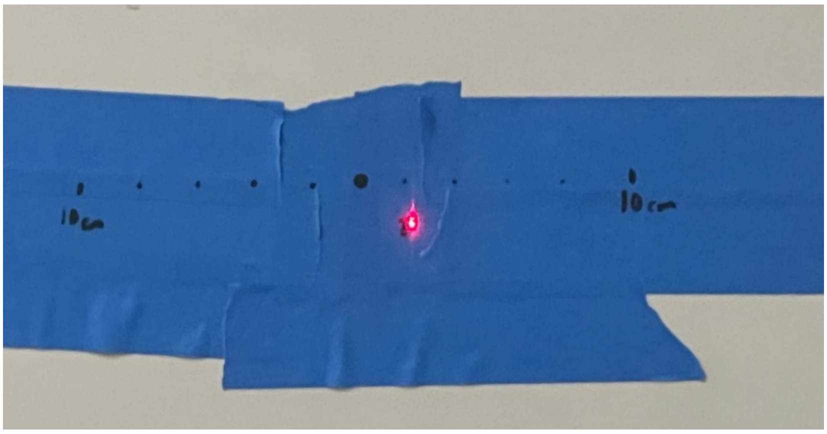

Figure 5.21: The point of the laser measurer reflected onto the back wall during the 12 mm outlet diameter testing.

From further relationships between these variables, the actual displacement of the tank could then be found. As the reflected beams are at the same angle, due to having the same angle of incidence, they are similar triangles and can thus be used to determine the horizontal displacement for each scenario. The difference in horizontal positions between the initial and maximum displacement positions is the same as the actual displacement of the tank.

Actual displacement $= 3.101\,cos(55.2°) - 3.089cos(55.2°) = 6.85\,mm$

As this displacement was considerably higher than the expected value of 3.14 mm, there were evidently several errors present in the experiment. The experiment was then redone with the 8 mm outlet nozzle attached, to investigate whether the outlet had caused the discrepancy between the expected and measured values.

From the repeated experiment, the distance from the laser to the wall via the mirror was the same, 3.089 m, however the distance to the wall during maximum displacement was slightly less than in the trial without the 8 mm nozzle. This value was measured to be 3.099 m. The difference between the measured values along the wall as seen in Figure 5.22 was found to be 1.8 cm, slightly less than the previous trial. Using these new measured values the angle of reflection for this trial was calculated as 56.3°. This value is similar to the previously calculated value, however it should be expected to remain the same, as the angle between the tank and laser remained unchanged. This difference was attributed to the tank deviating slightly from its original position throughout the period of both trials.
From a continuation of the calculations done for the previous trial, the actual displacement was found.

Actual displacement $= 3.099\,cos(56.3°) - 3.089cos(56.3°) = 5.55\,mm$

This displacement was smaller than the previous trial, and closer to the expected value from theoretical calculations, however as it was still larger than expected there were several factors affecting the results of both trials. Some of the factors that may have been present and affected the results are failure to allow for the tank to move in a single dimension, air flow, imperfections with the fishing line, and an unbalanced mass distribution of the tank.

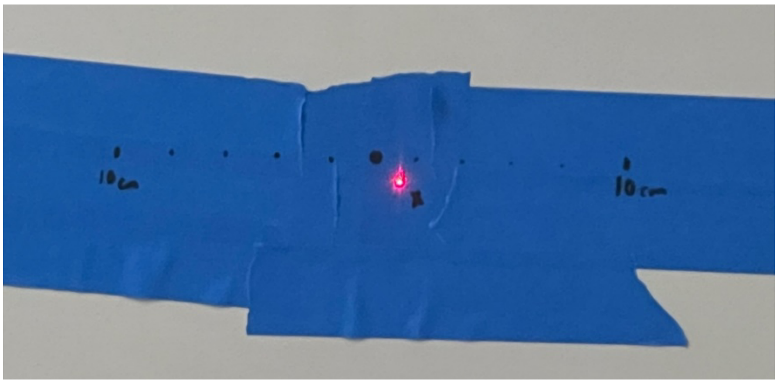

Figure 5.22: The point of the laser measurer reflected onto the back wall during the 8 mm outlet diameter testing.

5.2.6 Discussion

The method that was selected to suspend the tank was chosen in order to prevent friction from affecting the results by as much as possible. However, the fact that the tank oscillated in multiple directions rather than move along a single axis was not considered with too much concern. This was not thought to be as significant of an issue as it turned out to be because the expected motion was very minimal, and so it was unknown whether there would be any movement at all, thus it was vital to reduce frictional losses by as much as possible in order to obtain results that could demonstrate the force caused by an off-centre leak. From the results, it is clear that there is some effect, however, to completely understand just how significant this is, the experimental setup must be optimised further, to completely isolate the motion of the tank solely due to the leak.

Air flow disturbances were another factor that was minimised as much as possible, however it is likely still had some effect on the results. During the experiment, care was taken to ensure that all doors were closed and movement by people in the room was as minimal as possible, however there were certain contributions that were unavoidable. For example, for an ideal setup, the experiment could be conducted completely remotely in a negatively pressurised room, with remotely operated cameras to record the results. This would require almost no physical interactions with the setup of the experiment, allowing for the experiment coordinators to be occupied in a different room, where they cannot interfere with the results. The negatively pressurized room would also serve to reduce air flow in the room, minimising the effect of air resistance and other behaviours associated with conducting an experiment in standard atmospheric conditions. Although this setup would be ideal to eliminate air flow as a factor on the results, this was not achievable due to primarily financial and accessibility constraints. Conducting the experiment in a setup such as this would be costly and difficult to acquire, especially with the limited budget provided to cover the entirety of the project, though for further work on this topic availability to such a setup would be ideal for obtaining more accurate results.

Imperfections within the fishing line also may have affected the accuracy of the results. Any slight differences in lengths of line supporting the tank and unequal tension or elasticity within the line may have caused unbalanced motion. Attempts were made during the experiment to keep this as similar as possible, through using the same reel of fishing line, and cutting equal lengths of line. Despite these attempts, the line was noticed to be of different lengths once it was threaded through the tank and tied together. This resulted in the tank not sitting as perfectly level as intended, which may have altered the results slightly.

Another factor that influenced the validity of the results was that the centre of mass was not located at the centre of the tank. This was primarily due to the components attached to the tank in order to achieve remote flow such as the smart water valve, as they were only located on one side of the tank, this side was heavier, and thus the centre of gravity was shifted closer towards this side. As the original basis for the problem assumed that the mass distribution of the tank was balanced, all reference to the centre of the tank was due to the centre of gravity also being in line with the centre of gravity. As the displacement was found to be optimal as the distance from the hole to the centre was increased, any change in the centre of gravity would also affect the behaviour of the tank. For the experiment outlined throughout this report, the centre of gravity was shifted closer to the hole, reducing the length (from the hole to the centre of gravity) further than intended. Additionally, the water initially contributes to a majority of the mass of the total tank, but reduces this contribution as it empties, until the mass is solely dependent on the mass of the dry tank. This shifting centre of gravity further complicates the calculations associated with the problem and obscures the results expected. For future work on this project, it would be recommended to investigate the effect of a counterweight of similar mass to the smart valve and associated components, attached to the tank at an equal but opposite distance from the centre of the tank. Although this would increase the mass of the tank and would be expected to result in a smaller maximum displacement, the benefit to having a balanced mass distribution may achieve more desired results.

Several complications arose throughout the project that limited the number of tests that were able to be completed with the experimental model. The first of which, resulted in the scope of the project narrowing. As the initial intention for the project was to also investigate the effect of different parameters by constructing multiple physical tank models, but the project group size was decreased halfway into the timeline, resulting in the budget available also decreasing. At this point it was decided to focus on designing and building a single model that is optimised for maximum displacement, rather than constructing several smaller models to investigate the effect of different parameters. This option was chosen as this was determined to be the best method of producing experimental results, as there was a possibility that the motion of the smaller tanks would be even harder to detect or may not even move at all. Further complications arose during the initial testing of the tank. As the tank was being filled with water, to test if the setup could support the weight of the water the line used to suspend the tank snapped. Fortunately, as the tank fell a group member managed to safely catch it, preventing any catastrophic damage. However, the tank was only just over half full of water, significantly less than the intended capacity. For this reason, a stronger and thicker fishing line was used for subsequent testing, to allow for the tank to be filled up to its intended height. Finally, a rather catastrophic complication occurred following the second experimental trial. Upon completion of this trial, the tank was placed in a position that was thought to be secure, however it was unintendedly knocked over, falling onto the floor. This fall resulted in the tap timer ceasing to function properly within future tests. At the point at which this occurred, much of the budget had already been spent, primarily on materials used for the construction of the tank model, so a replacement smart valve could not be purchased. This complication resulted in only two tests being completed, much less than the intended scope of the project, meaning that the experimental procedure could not be adjusted to increase the accuracy of the results.

6. Recommendations for Future Work

The leaky tank mystery has now been tested experimentally; however, this does not mean that this mystery is no more. When solving the leaky tank mystery in this project, two main avenues were investigated, the theoretical aspect through use of mathematical formulae and simulative software, and also an experimental route. The theoretical path explored the behaviour of the tank based on a number of different parameters such as dimensions and location of the hole. From this, a model of the tank was created on COMSOL to simulate the movement of the tank. However, since the proper licensing requirements were not acquired, the results computed were based on a set of predetermined equations used to manually map the motion of the tank.

This was not what the intended use of COMSOL was for, but rather for it to compute the expected motion of the tank based on the water flow through the outlet. As such, it is recommended that future work explores the different physics packages available in COMSOL to accurately measure the displacement of the leaky tank, rather than basing the movement on predefined displacements already evaluated. Furthermore, the leaky tank model could be replicated on numerous other multiphysics simulative software such as Siemens Simcenter STAR-CCM+, OpenFOAM and Autodesk CFD. Therefore, simulation of the leaky tank model across many programs could be used to validate any results generated. A different type of model could also be designed to prove the motion of the leaky tank car, such as one that precisely imitates the original leaky tank mystery, a tank car that rests four wheels upon a frictionless surface.

Although the experimental testing of the leaky tank model did yield a set of results, a number of questions still remain unanswered. The results gained through experimental means did indicate that the water being released from the tank does affect the motion of the tank. This is indicated by the increase in displacement measured between the testing with and without the 8 mm outlet nozzle attached. Aforementioned in the discussion of these results, presence of errors was indicated, therefore future work could be completed to mitigate these errors. The continual swaying of the tank in all dimensions, was one of these errors that caused variation from the expected results. Future testing that could minimise the sway of the tank includes the testing being performed within a negatively pressurised room. In doing this, any airflow which would typically cause swaying to occur could be eliminated, thus providing a more accurate depiction of how the water being released causes the tank to behave. However, to truly eliminate the effects of sway in multiple dimensions, future projects should aim to isolate motion of the tank in only one dimension. An issue with the suspended tank prototype designed is that motion in only one dimension cannot be isolated, since the tank will always have a component of vertical motion when it is in motion. Therefore, this would require a new model to be designed.

A set of new prototypes could also be designed to test the same principles tested in this project. An example of such prototypes includes a leaky tank car upon an air table used to minimise friction, however it would be imperative to set up an effective drainage system as to not expose the air table to the water released. A large variety of parameters could also be experimented with to observe their effect on the motion of the tank. These parameters include the fluid tested, shape of the tank, mass of the tank, or even inclusion of a counterweight to account for the weight of the water release mechanism. Using a combination of information about this mystery and the work completed within this project, future projects can tackle all different aspects of the leaky tank mystery.

7. Conclusion

The investigation into the leaky tank mystery delivered fascinating results that allowed room for discussion and analysis. OB1 successfully identified the design parameters that had the greatest effect on displacement of the tank, which allowed for a suitable model to be produced. Although this model produced was intended to maximise displacement, several limiting factors were encountered, notably the size of the tank was limited by the thickness and strength of the selected materials. The materials used were the most suitable for the budget of the project, however a larger budget could allow for further testing on larger tank sizes and several models tested. Following the determination of dimensions for the leaky tank model within the parameter analysis, a COMSOL model was able to be designed. Although, the idealised COMSOL packages to be used were locked behind licensing issues, the displacement of tank was instead tested for using predetermined expressions of velocity. Using this method, it was expected that the tank would displace a maximum of 3.144 mm, however the computed solutions from the COMSOL method instead stated that the velocity would be zero, as such indicating a stationary tank. Thus, this suggests an error is present within the COMSOL setup.

Although the experimental results of OB2 were not validated by the findings in OB1, there was evidence of the leaky tank experiencing a greater displacement than expected. This finding provides indication to the potential of future work on the topic, as there was found to be a measurable disturbance, which could be further refined to gain a greater understanding on optimising the problem. Although several factors were present that affected the experimental results, these were identified and addressed, with suggestions for minimising their effect on the tank outlined.

Although complications throughout the project limited the scope of the project, there appears to be substantial validity to the results to allow for further work, expanding the projecting towards the initial scope intended for the project. This includes conducting a greater number of trials, for a greater sample size and further refinement of the setup and method used to achieve the outcomes of OB2. Reduction of the group project size and budget also directly impacted the capability of the group's experimental testing procedures and prototypes. It should also be concluded that the results generated experimentally were found to have differed from the theoretical values, however both did agree that the motion of the tank was dependent on the design parameters and fluid flow properties.

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9. Appendix A:
Parameter Analysis Code

Table of Contents

Honours Project Tank Sizing

```
% Defining dimensions

% More width results in more displacement and more time to max
w = 0.3; % width = .3 m

% More height results in more displacement and more time to max
h = 0.2; % water height = 0.2 m

% Larger diameter results in shorter emptying time and slightly less
% displacement
D = 0.025; % diameter of hole = 2.5 cm

% More depth results in more displacement and more time to empty
z = 0.2; % depth = 0.2 m

% Distance to hole from centre. Maximum displacement at L=w/2
L = 0.633*w/2; % distance to hole from centre

% Thickness of acrylic. More thickness leads to less displacement.
th = 0.005; % thickness of acrylic = 0.5 cm

% mass of dry tank = 0.1 kg
m = 1180*(z*w*th+2*th*h*(w+z));

% Initial Conditions
t = 0; % time = 0 s

% initial vertical velocity of water exiting the tank
v0 = sqrt(2*9.81*h);

% Area of bottom of tank
A_c = w*z;
% Area of hole in bottom of tank
A_h = pi/4*D^2;
```

```
% Time for the tank to empty
t_e = A_c/A_h*sqrt(2*h/9.81)

mu = w*h*z/m;

% Initial velocity of the tank car
v_0 = -2*L/t_e*(mu/(1+mu))

% Initialising variables
max=0;
t_max = 0;

% Finding maximum displacement and time for max displacement
for t=0:250
    time = 1-t/t_e;
    x = -L*((mu/(1+mu))*t/t_e-.5*(log10((1+mu)/
(1+mu*(time)^2))-2*sqrt(mu)*(time)*(atan(180/pi*sqrt(mu))-(atan(180/
pi*sqrt(mu))*(time)))));
    if x < max
        max = x;
        t_max = t;
    end
end

% Printing values for maximum displacement and the time at which it
 occurs
max
t_max
```

Width Analysis

Defining dimensions

```
h = 0.2; % water height = 0.2 m
z = 0.2; % depth = 0.2 m
D = 0.025; % constant diameter of hole = 0.02 m
th = 0.005; % thickness of acrylic = 0.5 cm
g = 9.81; % gravitational constant

% Different widths
widths = [0.2, 0.3, 0.5, 1.0]; % varying widths in meters

% Prepare figure for plotting
figure;
hold on;
colors = ['r', 'g', 'b', 'k']; % colors for each width

% Loop over different widths and plot
for i = 1:length(widths)
    w = widths(i); % width of the tank
    L = 0.9 * w / 2; % distance to hole from center
```

```
    % Updated mass calculation
    m = 1180 * (z * w * th + 2 * th * h * (w + z)); % mass of dry tank

    A_c = w * z; % area of bottom of tank
    A_h = pi / 4 * D^2; % area of hole
    t_e = A_c / A_h * sqrt(2 * h / g); % time for the tank to empty
    mu = w * h * z / m; % constant for velocity calculation
    v_0 = -2 * L / t_e * (mu / (1 + mu)); % initial velocity of tank
 car

    % Time vector only up to t_e
    t_values = linspace(0, t_e, 1000); % time from 0 to t_e in 1000
 steps

    % Calculate displacement over time in millimeters
    x_values = zeros(size(t_values));
    for j = 1:length(t_values)
        t = t_values(j);
        time = 1 - t / t_e;
        x = -L * ((mu / (1 + mu)) * t / t_e - 0.5 * (log10((1 + mu) /
 (1 + mu * time^2)) - ...
            2 * sqrt(mu) * time * (atan(180/pi * sqrt(mu)) -
 (atan(180/pi * sqrt(mu)) * time))));
        x_values(j) = x * 1000; % convert meters to millimeters
    end

    % Plot the displacement for this width
    plot(t_values, x_values, colors(i), 'LineWidth', 2, 'DisplayName',
 sprintf('w = %.2f m', w)); % Added LineWidth for thicker lines
end

% Labels and legend
xlabel('Time (s)', 'FontSize', 14); % Adjusted font size
ylabel('Displacement (mm)', 'FontSize', 14); % Change label to mm,
 adjusted font size
title('Displacement vs Time for Different Tank Widths', 'FontSize',
 16); % Adjusted font size
legend show;

% Set line width for axes and add border
set(gca, 'LineWidth', 1.5); % Make axes lines thicker
box on; % Add a border around the plot area

hold off;
```

L Ratio Analysis

```
% Defining dimensions
w = 0.3; % constant width = 0.3 m
h = 0.2; % water height = 0.2 m
z = 0.2; % depth = 0.2 m
D = 0.025; % constant diameter of hole = 0.02 m
th = 0.005; % thickness of acrylic = 0.5 cm
```

```
g = 9.81; % gravitational constant
m = 1180 * (z * w * th + 2 * th * h * (w + z)); % mass of dry tank
A_c = w * z; % Area of bottom of tank

% Initial parameters
v0 = sqrt(2 * g * h); % initial vertical velocity of water exiting

% Range of L_ratios (ratios of L/w)
L_ratios = [0, 0.25, 0.5, 0.75, 1]; % different ratios for L/w

% Prepare figure for plotting
figure;
hold on;
colors = ['r', 'g', 'b', 'k', 'm']; % colors for each L/w ratio

% Loop over different L/w ratios and plot
for i = 1:length(L_ratios)
    L_ratio = L_ratios(i); % L as a ratio of width
    L = L_ratio * w / 2; % distance to hole from center based on the
 ratio

    % Area of hole remains constant
    A_h = pi / 4 * D^2; % area of hole
    t_e = A_c / A_h * sqrt(2 * h / g); % time for the tank to empty
    mu = w * h * z / m; % constant for velocity calculation
    v_0 = -2 * L / t_e * (mu / (1 + mu)); % initial velocity of tank
 car

    % Time vector only up to t_e
    t_values = linspace(0, t_e, 1000); % time from 0 to t_e in 1000
 steps

    % Calculate displacement over time in millimeters
    x_values = zeros(size(t_values));
    for j = 1:length(t_values)
        t = t_values(j);
        time = 1 - t / t_e;
        x = -L * ((mu / (1 + mu)) * t / t_e - 0.5 * (log10((1 + mu) /
 (1 + mu * time^2)) - ...
            2 * sqrt(mu) * time * (atan(180/pi * sqrt(mu)) -
 (atan(180/pi * sqrt(mu)) * time))));
        x_values(j) = x * 1000; % convert displacement to millimeters
    end

    % Plot the displacement for this L_ratio
    plot(t_values, x_values, colors(i), 'LineWidth', 2, 'DisplayName',
 sprintf('2L/w = %.2f', L_ratio)); % Added LineWidth for thicker lines
end

% Labels and legend
xlabel('Time (s)', 'FontSize', 14); % Adjusted font size
ylabel('Displacement (mm)', 'FontSize', 14); % Adjusted font size
title('Displacement vs Time for Different L/w Ratios', 'FontSize',
 16); % Adjusted font size
```

```
legend show;

% Set line width for axes and add border
set(gca, 'LineWidth', 1.5); % Make axes lines thicker
box on; % Add a border around the plot area

hold off;
```

Diameter Sizing Analysis

```
% Defining dimensions
w = 0.3; % width = 0.3 m
h = 0.2; % water height = 0.2 m
z = 0.2; % depth = 0.2 m
L = 0.633 * w / 2; % distance to hole from centre = 0.09 m
th = 0.005; % thickness of acrylic = 0.5 cm
m = 1180 * (z * w * th + 2 * th * h * (w + z)); % mass of dry tank
A_c = w * z; % Area of bottom of tank
g = 9.81; % gravitational constant

% Initial parameters
v0 = sqrt(2 * g * h); % initial vertical velocity of water exiting

% Different diameters
diameters = [0.01, 0.02, 0.03, 0.04]; % diameters in meters

% Prepare figure for plotting
figure;
hold on;
colors = ['r', 'g', 'b', 'k']; % colors for each diameter

% Loop over different diameters and plot
for i = 1:length(diameters)
    D = diameters(i); % diameter of hole
    A_h = pi / 4 * D^2; % area of hole
    t_e = A_c / A_h * sqrt(2 * h / g); % time for the tank to empty
    mu = w * h * z / m; % constant for velocity calculation
    v_0 = -2 * L / t_e * (mu / (1 + mu)); % initial velocity of tank
 car

    % Time vector only up to t_e
    t_values = linspace(0, t_e, 1000); % time from 0 to t_e in 1000
 steps

    % Calculate displacement over time
    x_values = zeros(size(t_values));
    for j = 1:length(t_values)
        t = t_values(j);
        time = 1 - t / t_e;
        x = -L * ((mu / (1 + mu)) * t / t_e - 0.5 * (log10((1 + mu) /
 (1 + mu * time^2)) - ...
            2 * sqrt(mu) * time * (atan(180/pi * sqrt(mu)) -
 (atan(180/pi * sqrt(mu)) * time))));
```

```
        x_values(j) = x*1000; % in millimeters
    end

    % Plot the displacement for this diameter
    plot(t_values, x_values, colors(i), 'LineWidth', 2, 'DisplayName',
 sprintf('D = %.2f m', D)); % Added LineWidth for thicker lines
end

% Labels and legend
xlabel('Time (s)', 'FontSize', 14); % Adjusted font size
ylabel('Displacement (mm)', 'FontSize', 14); % Adjusted font size
title('Displacement vs Time for Different Hole Diameters', 'FontSize',
 16); % Adjusted font size
legend show;

% Set line width for axes and add border
set(gca, 'LineWidth', 1.5); % Make axes lines thicker
box on; % Add a border around the plot area

hold off;
```

Height Analysis

```
% Defining dimensions
w = 0.3; % constant width = 0.3 m
z = 0.2; % constant depth = 0.2 m
D = 0.025; % constant diameter of hole = 0.02 m
th = 0.005; % thickness of acrylic = 0.5 cm
g = 9.81; % gravitational constant

% Different water heights
heights = [0.1, 0.2, 0.3, 0.5, 1.0]; % varying water heights in meters

% Prepare figure for plotting
figure;
hold on;
colors = ['r', 'g', 'b', 'k', 'm']; % colors for each height

% Loop over different heights and plot
for i = 1:length(heights)
    h = heights(i); % height of the water
    L = 0.633 * w / 2; % distance to hole from center

    % Updated mass calculation
    m = 1180 * (z * w * th + 2 * th * h * (w + z)); % mass of dry tank

    A_c = w * z; % area of bottom of tank
    A_h = pi / 4 * D^2; % area of hole
    t_e = A_c / A_h * sqrt(2 * h / g); % time for the tank to empty
    mu = w * h * z / m; % constant for velocity calculation
    v_0 = -2 * L / t_e * (mu / (1 + mu)); % initial velocity of tank
 car
```

```
    % Time vector only up to t_e
    t_values = linspace(0, t_e, 1000); % time from 0 to t_e in 1000
 steps

    % Calculate displacement over time in millimeters
    x_values = zeros(size(t_values));
    for j = 1:length(t_values)
        t = t_values(j);
        time = 1 - t / t_e;
        x = -L * ((mu / (1 + mu)) * t / t_e - 0.5 * (log10((1 + mu) /
 (1 + mu * time^2)) - ...
            2 * sqrt(mu) * time * (atan(180/pi * sqrt(mu)) -
 (atan(180/pi * sqrt(mu)) * time))));
        x_values(j) = x * 1000; % convert meters to millimeters
    end

    % Plot the displacement for this height
    plot(t_values, x_values, colors(i), 'LineWidth', 2, 'DisplayName',
 sprintf('h = %.2f m', h));
end

% Labels and legend
xlabel('Time (s)', 'FontSize', 14);
ylabel('Displacement (mm)', 'FontSize', 14); % Change label to mm
title('Displacement vs Time for Different Water Heights', 'FontSize',
 16);
legend show;

% Set line width for axes and add border
set(gca, 'LineWidth', 1.5); % Make axes lines thicker
box on; % Add a border around the plot area

hold off;
```

Depth Analysis

```
% Defining dimensions
h = 0.2; % water height = 0.2 m
w = 0.3; % width = 0.3 m
D = 0.025; % constant diameter of hole = 0.025 m
th = 0.005; % thickness of acrylic = 0.5 cm
g = 9.81; % gravitational constant

% Different widths
depths = [0.1, 0.2, 0.3, 0.5, 1.0]; % varying depth in meters

% Prepare figure for plotting
figure;
hold on;
colors = ['r', 'g', 'b', 'k', 'm']; % colors for each depth

% Loop over different depths and plot
for i = 1:length(depths)
```

```
    z = depths(i); % depth of the tank
    L = 0.633 * w / 2; % distance to hole from center

    % Updated mass calculation
    m = 1180 * (z * w * th + 2 * th * h * (w + z)); % mass of dry tank

    A_c = w * z; % area of bottom of tank
    A_h = pi / 4 * D^2; % area of hole
    t_e = A_c / A_h * sqrt(2 * h / g); % time for the tank to empty
    mu = w * h * z / m; % constant for velocity calculation
    v_0 = -2 * L / t_e * (mu / (1 + mu)); % initial velocity of tank
 car

    % Time vector only up to t_e
    t_values = linspace(0, t_e, 1000); % time from 0 to t_e in 1000
 steps

    % Calculate displacement over time in millimeters
    x_values = zeros(size(t_values));
    for j = 1:length(t_values)
        t = t_values(j);
        time = 1 - t / t_e;
        x = -L * ((mu / (1 + mu)) * t / t_e - 0.5 * (log10((1 + mu) /
 (1 + mu * time^2)) - ...
             2 * sqrt(mu) * time * (atan(180/pi * sqrt(mu)) -
 (atan(180/pi * sqrt(mu)) * time))));
        x_values(j) = x * 1000; % convert meters to millimeters
    end

    % Plot the displacement for this width
    plot(t_values, x_values, colors(i), 'LineWidth', 2, 'DisplayName',
 sprintf('z = %.2f m', z)); % Added LineWidth for thicker lines
end

% Labels and legend
xlabel('Time (s)', 'FontSize', 14); % Adjusted font size
ylabel('Displacement (mm)', 'FontSize', 14); % Change label to mm,
 adjusted font size
title('Displacement vs Time for Different Tank Depths', 'FontSize',
 16); % Adjusted font size
legend show;

% Set line width for axes and add border
set(gca, 'LineWidth', 1.5); % Make axes lines thicker
box on; % Add a border around the plot area

hold off;
```

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