Physical-layer encryption on the public internet: a stochastic approach to the Kish-Sethuraman cipher

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Abstract

While information-theoretic security is often associated with the one-time pad and quantum key distribution, noisy transport media leave room for classical techniques and even covert operation. Transit times across the public internet exhibit a degree of randomness, and cannot be determined noiselessly by an eavesdropper. We demonstrate the use of these measurements for information-theoretically secure communication over the public internet.

1   Introduction

Throughout history claims have abounded of supposedly unbreakable codes and ciphers, however it was not until the 20th century that the mathematical underpinnings of cryptology gave them any degree of credibility.

The first truly unbreakable cipher was the one-time-pad [1], invented in the United States in 1918, but independently developed and first put into practice by the German Foreign Office in the early 1920s. This scheme, however, is hampered by the need to distribute a key to the message recipient of the same size as the message and in perfect secrecy. This onerous key distribution arrangement ruled it out for all but the most critical applications.

In recent years BB84, introduced by Bennett and Brassard [2], provides security by encoding data in one of two non-orthogonal bases, chosen at random for each photon. The measurement basis is also chosen at random. After the measurement has taken place, the two parties reveal their chosen bases and

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discard those bits that were not measured in the correct manner. As an eavesdropper does not know in advance which basis was used, she cannot reliably copy its polarisation state and so any eavesdropping will manifest itself as an increase in the bit-error rate (BER). This allows eavesdropping to be detected by the two legitimate parties.

Later, Maurer [3] considered a more general case, letting the sender ('Alice' hereafter), recipient (Bob), and eavesdropper (Eve) each perform measurements X, Y, and Z respectively. He demonstrated that secure information exchange could occur (subject to some constraints) with an arbitrary joint distribution P_{XYZ}, reopening the door to information-theoretically secure communication over a classical channel in some circumstances.

In 2004, Kish and Sethuraman proposed a classical protocol [4] based upon commutative one-way encryption operators. If the sender and recipient each apply a layer of encryption to a message, then commutativity allows the sender to reverse her own operation and so produce a ciphertext with key known only to the recipient. In order to overcome the information-theoretic limits given by Maurer [3] we have attempted to relax the constraints of the protocol [4] by using the random transit times of the internet as encryption operations—this imperfect channel allows secure communication without resort to the one-time pad.

2   Round-trip times as a source of randomness

The essence of key distribution is to provide two endpoints with a shared secret that remains unknown to eavesdroppers. A subtle point is that the endpoints need not generate a secret and share it themselves, but could instead obtain it from elsewhere, provided an eavesdropper cannot do the same without error.

One such source of random data is the transit time between two internet-connected terminals. If Alice and Bob rally information packets back and forth via the internet, the time of each transit is a quantity common to the measurements of both, but measurable only with the addition of noise from the return trip (see Figure 1). An eavesdropper will suffer the same problem, however her noise will differ from that of Alice and Bob. This difference prevents her from taking advantage of the error correction performed by Alice and Bob during the information reconciliation [5] (IR) phase of processing, which discards bits likely to be incorrect, much like in the quantum protocol.

We propose to extract random bits from the round-trip times by finding their median and declaring those times greater than the median to be a one, and those less to be a zero. With only one bit per round-trip, we avoid the problem that errors are more likely to fall into a adjacent quantisation bins and so create correlations between bits.

While the distribution of round-trip times is actually quite skewed [6], we attempt to illustrate the technique theoretically by assuming transit times to be normally distributed and computing an upper limit on the key rate.
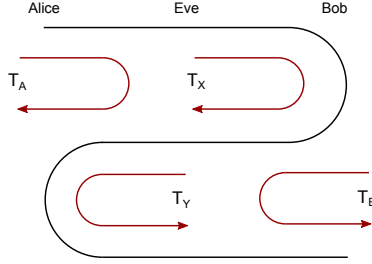
Figure 1: Consecutive round-trip measurements, where Bob's response to Alice forms a request for another measurement. The transit time of this intermediate transmission contributes to the measurements of Alice, Bob, and Eve, and so provides a source of mutual information.

2.1 The mutual information rate between endpoints

Let us denote the three packet transit times from Figure 1 as T_1, T_2, and T_3 respectively. Then, $T_A = T_1 + T_2$, $T_B = T_2 + T_3$. Suppose the three $T_i \sim \mathcal{N}(0, 1)$. Then, as the distribution (and so the channel) is symmetric, we may calculate the bit error rate as

$$1 - P[T_B < 0 | T_A < 0] = 1 - P[T_2 + T_3 < 0 \mid T_1 + T_2 < 0] \tag{1}$$

$$= 1 - \int_{-\infty}^{+\infty} 2\phi(t_2) P[T_3 < -t_2 \cap T_1 < -t_2] \, dt_2 \tag{2}$$

$$= 1 - \int_{-\infty}^{+\infty} 2\phi(t_2) \Phi^2(-t_2) \, dt_2 \tag{3}$$

$$= 1 - \int_0^1 2u^2 \, du \tag{4}$$

$$= \frac{1}{3}, \tag{5}$$

where $\phi(t)$ and $\Phi(t)$ are the probability density and cumulative probability functions respectively of the normal distribution function. It should be noted that the derivation above holds for any zero-median symmetric distribution rather than just for the normal distribution.

This BER of 1/3 corresponds to a channel capacity of 0.08 bits/measurement, suggesting that the achievable key rate with this technique may be too low for direct use as a one-time pad.

2.2 Limitations

Despite the allure of information-theoretically secure key agreement without specialised hardware, this method is not perfect and is necessarily dependent on the eavesdropper's inability to timestamp packets with perfect accuracy. This limits its use where an eavesdropper can timestamp packets on the link directly.

To illustrate this point, we make use of Maurer's upper bound [3] on the secrecy rate, the maximum rate at which information can be transmitted securely,

$$S(X; Y || Z) \leq I(X; Y).$$

This states that the rate at which secure communication may take place is limited to the mutual information of the two endpoints. The effect of this statement is that a protocol, no matter how clever, cannot provide secrecy using only independent random number generators at each end.

In order to demonstrate the relevance of this inequality, imagine that Eve can timestamp Alice's transmissions without error. Then, Alice cannot gain an advantage over Eve, who has the same information as Alice, and so secrecy is impossible.

Now imagine that we have placed a router between Alice and Bob. This introduces some randomness, but Alice could achieve the same effect by simply adding a random delay to her transmissions; that is to say, it is as though she used a random unshared key. As discussed, this cannot form the basis for a secure system. Therefore, if Eve can measure without noise, information-theoretic security is not possible.

However, there are many cases in which this is not true. If an eavesdropper simply has copies of all traffic forwarded to them (such as provided by port mirroring), then routing delays and packet reordering provide the necessary source of noise [7]. If a standard PC is acting as a man-in-the-middle, uncertainty in the timing routines of its operating system provide an additional source of noise. These factors allow the system to provide security, especially against unsophisticated eavesdroppers using only commodity network hardware that does not provide hardware timestamping facilities.

3 Experimental Round-Trip Measurements

In order to demonstrate this technique, we constructed a test system to determine the performance of the method in the presence of an eavesdropper. The test system rallied UDP packets back and forth along a chain of hosts (see Figure 2), with the time of each arrival being timestamped. While the timescales were not synchronised, this information is sufficient to determine the various round-trip times.

The effect of information reconciliation is shown in Figure 3. While the BER of the eavesdropper falls at first, it soon reaches a minimum value of around 2%. This demonstrates that some nonzero secrecy rate is achievable via privacy amplification.

4 Demonstration System

We have implemented the described protocol, which has been successfully operated over the internet. Round-trip times are measured using UDP packets,
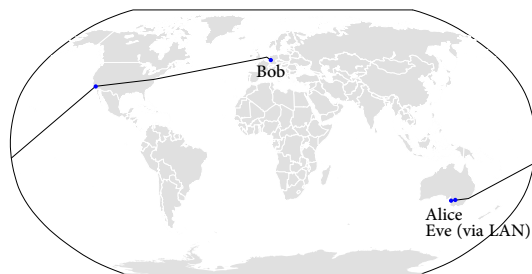
Figure 2: The constructed communications link. Alice is located in Adelaide, Australia, in the same room as Eve, to whom she is connected via the local network. The packet is then forwarded to through a relay in Los Angeles, and finally to Bob in Frankfurt. Alice sends a packet to Eve, who forwards it to Los Angeles, and finally to Bob in Frankfurt. The packet is then returned. At each step the arrival of the packet is timestamped, and the packet finally returned to Alice contains a time of arrival for Bob, and two for each intermediate note. The head of the chain in Adelaide, representing Alice, sends a packet which is timestamped at each of the three other hosts. The demonstration system described later does not transmit timestamps over the network, allowing each node to determine only its own round-trip time.

whose times of transmission and receipt are determined using operating system routines. If a timeout occurs, due to a dropped packet for instance, the trip is marked as such and dropped during the reconciliation process. Information reconciliation is performed using the bit-pair iteration [3] protocol, each end-point executing identical code in lock-step (with the exception of a network abstraction layer).

Parameters for the information reconciliation and privacy amplification are determined automatically. A lower bound on eavesdropper BER is given as a parameter, and so their channel capacity is computed and thus the amount of information that they hold. From this, a hashing function is chosen—the sum of some number of bits modulo two—that will discard sufficient information to eliminate the eavesdropper's knowledge of the secret key. As this process will increase the BER of the legitimate parties also, the target BER for the information reconciliation is reduced to compensate.

The BER of the channel is estimated using the error rate of the parity bits. A 2σ Agresti-Coull [8] confidence interval is constructed and back-propagated through the binomial probability mass function [9], yielding a confidence interval for the BER of the underlying channel. Then, the BER of the parity-checked output of each iteration can be computed recursively in order to determine an interval containing the required number of IR iterations. When this interval has been reduced to two possible values, we take the larger of the values and continue with the complete reconciliation process.

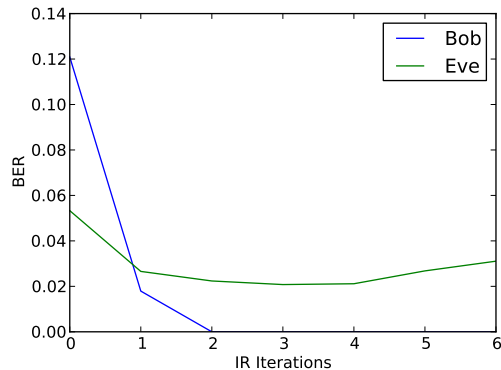We succeed in generating keys at a rate of 13 bits/minute over the link

Figure 3: The effect of bit-pair iteration upon the BER between Alice/Bob and Alice/Eve. This measurement used approximately 30,000 round-trips AU-US-EU, with the eavesdropper chosen to be the node in the same room as the sender. While the BER of the eavesdropper is improved slightly, it is not reduced to zero, which is evidence that there is sufficient measurement noise to allow secure communication.

shown in Figure 2, the lower bound on the eavesdropper BER set at 10^{-2}, based on the results shown in Figure 3. The 400 ms round-trip time makes the test relatively pessimistic by terrestrial standards, and greater key rates are potentially achievable across shorter distances.

5 Conclusion

We have considered the use of packet timing over the internet for information-theoretically secure key agreement, and demonstrated the feasibility of the technique experimentally. We have developed a practical implementation of the method that is capable of generating shared keys in real-time despite the assumption of an eavesdropper BER equivalent to a man-in-the-middle attack from within the local network.

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